Homework: pg 400 #1, 2, 3, 4-6(ace), 8ace, *9ab (see pg 398), 11a, 13, (16)

Compound Angle Formulas

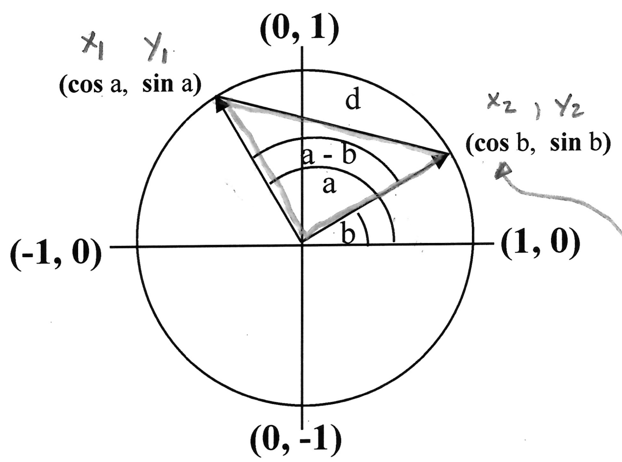
Key Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin(-\theta) = -\sin(\theta)$ $\qquad$ $\cos(-\theta) = \cos(\theta)$

$\sin\left(\theta - \dfrac{\pi}{2}\right) = -\cos(\theta)$ $\qquad$ $\cos\left(\theta - \dfrac{\pi}{2}\right) = \sin(\theta)$

Consider the following terminal arms on a unit circle



$\dfrac{\cos\theta}{1} = \dfrac{x}{r}$

$x = r\cos\theta$

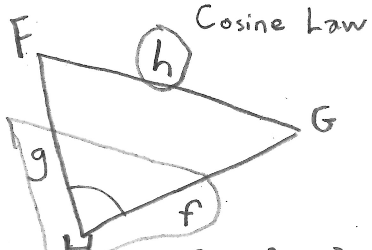
$x = 1\cos b$

$\boxed{x = \cos b}$

Activity

a) Determine the square of the length of line d using the cosine law.

Cosine Law



$h^2 = f^2 + g^2 - 2fg\cos H$

$d^2 = 1^2 + 1^2 - 2(1)(1)\cos(a-b)$

① $d^2 = 2 - 2\cos(a-b)$

b) Determine the square of the length of line d using Pythagorean Theorem; $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

$(\cos b - \cos a)^2$

$= (\cos b - \cos a)(\cos b - \cos a)$

$= \cos^2 b - 2\cos a\cos b + \cos^2 a$

$d^2 = (\cos b - \cos a)^2 + (\sin b - \sin a)^2$

$= \cos^2 b - 2\cos a\cos b + \cos^2 a + \sin^2 b - 2\sin a\sin b + \sin^2 a$

$= \underbrace{\sin^2 a + \cos^2 a}_{1} + \underbrace{\sin^2 b + \cos^2 b}_{1} - 2\sin a\sin b - 2\cos a\cos b$

② $d^2 = 2 - 2\sin a\sin b - 2\cos a\cos b$

c) Substitute the answer from a) into the answer for b)

sub 1 into 2

$$\cancel{1}-2\cos(a-b)=\cancel{1}-2\sin a\sin b-2\cos a\cos b$$

$$\frac{-2\cos(a-b)}{-2}=\frac{-\cancel{2}\sin a\sin b}{-\cancel{2}}\;\frac{-\cancel{2}\cos a\cos b}{-\cancel{2}}$$

$$\boxed{\cos(a-b)=\cos a\cos b+\sin a\sin b}$$

d) Create an equation for $\cos(a+b)$ by replacing the variable 'b' with '-b'.

$$\cos[a-(-b)]=\cos a\cos(-b)+\sin a\sin(-b)$$

$$\cos(a+b)=\cos a\cos b+\sin a(-\sin b)$$

$$\boxed{\cos(a+b)=\cos a\cos b-\sin a\sin b}$$

e) Create an equation for $\sin(a+b)$ using the identity $\sin(\theta)=\cos\left(\theta-\frac{\pi}{2}\right)$.

$$\sin(a+b)=\cos\left[(a+b)-\frac{\pi}{2}\right]$$

$$\left.\begin{aligned}&(a+b)-c\\&=a+b-c\\&=a+(b-c)\end{aligned}\right\}\longrightarrow\ =\cos\left[a+\left(b-\frac{\pi}{2}\right)\right]$$

$$=\cos a\cos\left(b-\frac{\pi}{2}\right)-\sin a\sin\left(b-\frac{\pi}{2}\right)$$

$$=\cos a\sin b-\sin a(-\cos b)$$

$$\boxed{\sin(a+b)=\sin a\cos b+\cos a\sin b}$$

f) Create an equation for $\sin(a-b)$ by replacing the variable 'b' with '-b'.

$$\sin[a+(-b)]=\sin a\cos(-b)+\cos a\sin(-b)$$

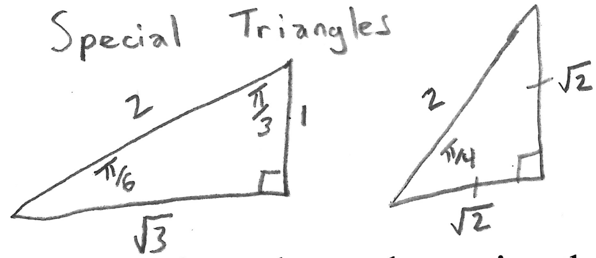
$$\boxed{\sin(a-b)=\sin a\cos b-\cos a\sin b}$$

g) Compound angle formulas for $\tan(a+b)$ and $\tan(a-b)$ can be created using the quotient identity: $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$.

In doing so, we get...

$$\boxed{\begin{aligned}\tan(a+b)&=\frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}\\ \tan(a-b)&=\frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}\end{aligned}}$$

Special Triangles



Example 1

Using the compound angle formulas to determine the exact value of the following:

a) $\sin\left(\frac{5\pi}{12}\right)$ (75°) $= \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$ (30°, 45°)

$= \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2} + \sqrt{6}}{4}$

b) $\cos\left(\frac{\pi}{12}\right)$ (15°) $= \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6} + \sqrt{2}}{4}$

Example 2

Simplify the following: $\cos\left(\frac{13\pi}{12}\right)\cos\left(\frac{11\pi}{12}\right) + \sin\left(\frac{13\pi}{12}\right)\sin\left(\frac{11\pi}{12}\right)$ (a, b)

$= \cos\left(\frac{13\pi}{12} - \frac{11\pi}{12}\right)$

$= \cos\left(\frac{2\pi}{12}\right)$

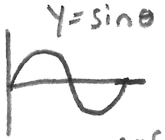
$= \cos\left(\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2}$

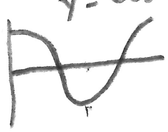
Example 3

Use compound angle formulas to show that $\cos(\theta + \pi) = \cos(\theta - \pi)$.

y = sinθ



y = cosθ



L.S. $= \cos(\theta + \pi)$

$= \cos\theta\cos\pi - \sin\theta\sin\pi$

$= \cos\theta(-1) - \sin\theta(0)$

$= -\cos\theta$

R.S. $= \cos(\theta - \pi)$

$= \cos\theta\cos\pi + \sin\theta\sin\pi$

$= \cos\theta(-1) + \sin\theta(0)$

$= -\cos\theta$

L.S. = R.S.

QED (Quod Erat Demonstratum)