Characteristics of Polynomial Functions

Polynomial functions have common features depending on the sign of the leading coefficient and the degree.

Leading Coefficient - the coefficient of the term with the highest degree in a polynomial; usually it is the first coefficient.

lead. coeff.

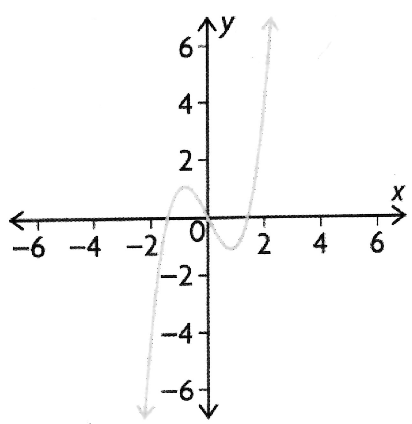
$y = -3x^5 + 2x^2 - 1$

$y = 2x^3 + 8x^5 - 7$

lead Coeff.

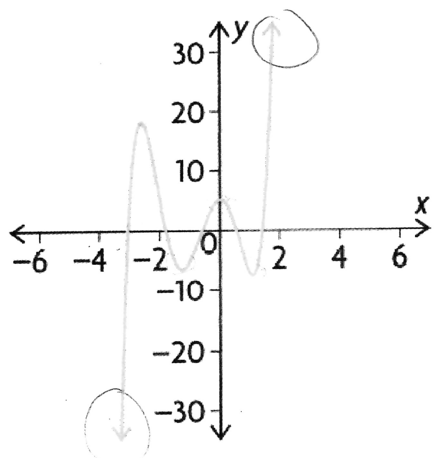
1. Examine the following functions and state their degree.

a) $f(x) = x^3 - 2x$



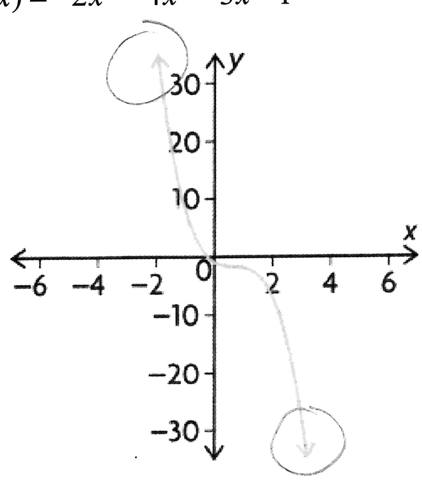
degree = 3

b) $f(x) = 2x^5 + 7x^4 - 3x^3 - 18x^2 + 5$



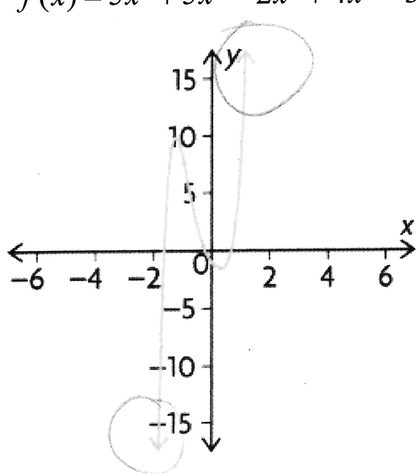
degree = 5

c) $f(x) = -2x^3 - 4x^2 - 3x - 1$



degree = 3

d) $f(x) = 5x^5 + 5x^4 - 2x^3 + 4x^2 - 3x$



degree = 5

All of the functions above have an odd degree.

Key Features of Odd Degree Functions

End behaviours (think of a line)

- If the leading coefficient is positive, then the function extends from the 3rd quadrant to the 1st quadrant; ie as $x \to -\infty, y \to -\infty$ and as $x \to \infty, y \to \infty$.
- If the leading coefficient is negative, then the function extends from the 2nd quadrant to 4th quadrant; ie as $x \to -\infty, y \to \infty$ and as $x \to \infty, y \to -\infty$.

Turning Points n → degree

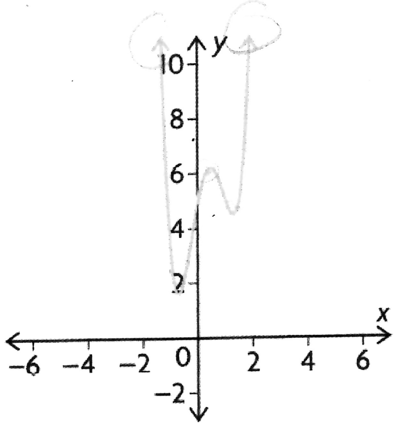
- These polynomials will have at most "n - 1" turning points; notice in 'd)' that there are only 2 turning points even though the function is of degree 5.

Number of Zeroes (x-intercepts)

- They will have at least one x-intercept with a maximum of 'n' x-intercepts.

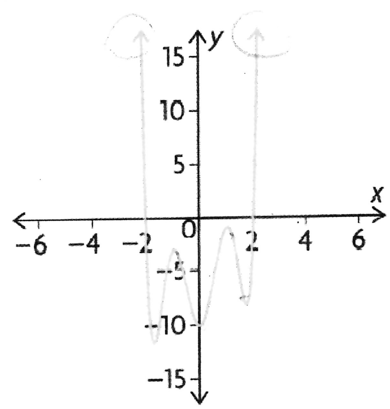
2. Examine the following functions and state their degree.

a) $f(x) = 3x^4 - 4x^3 - 4x^2 + 5x + 5$



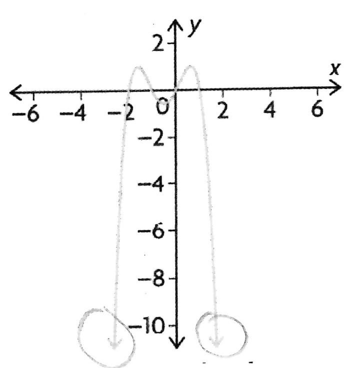
degree = 4

b) $f(x) = 2x^6 - 12x^4 + 18x^2 + x - 10$



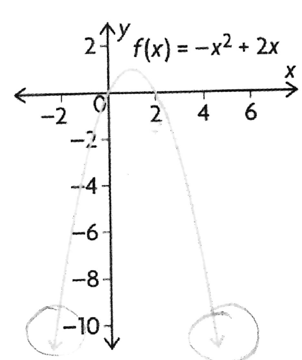
degree = 6

c) $f(x) = -x^4 - 2x^3 + x^2 + 2x$



degree = 4

d) $f(x) = -x^2 + 2x$



degree = 2

All of the functions above have an even degree.

Key Features of Even Degree Functions

End behaviours

- If the leading coefficient is positive, then the function extends from the 2nd quadrant to the 1st quadrant; ie as $x \to -\infty, y \to \infty$ and as $x \to \infty, y \to \infty$.
- If the leading coefficient is negative, then the function extends from the 3rd quadrant to 4th quadrant; ie as $x \to -\infty, y \to -\infty$ and as $x \to \infty, y \to -\infty$.

Turning Points

- These polynomials will have at most "n - 1" turning points. They will have at least one turning point.

Number of Zeroes (x-intercepts)

- They may not have any x-intercepts but can have a maximum of 'n' x-intercepts.

Symmetry of Polynomial Functions

Polynomial functions can have odd, even or no symmetry. If a polynomial does have symmetry, it tends to follow the degree. ie; If a quartic function (degree 4) has symmetry then it will be even since the degree is even, but not all quartics have symmetry. Ex; $f(x) = 4x^4 - 2x^2 + x$

*****CAREFUL!! While the symmetry and degree of a function are similar, they are not necessarily the same concept. ***** book (odd or even ⇒ symmetry)

Practice

Given the followig functions, complete the tables below.

a) $f(x) = 2x^5 - 3x^3 + x + 8$

lead. coeff.

Degree	5
Even or Odd Degree	Odd
Sign of Leading Coefficient	Positive
Max # of Turning Points	4
Max # of x-ints	5
End Behaviours	

as $x \to -\infty$, $y \to -\infty$
as $x \to +\infty$, $y \to \infty$

b) $f(x) = 1x^4 + 6x^3 + 2x$

lead. coeff.

Degree	4
Even or Odd Degree	Even
Sign of Leading Coefficient	Positive
Max # of Turning Points	3
Max # of x-ints	4
End Behaviours	

as $x \to -\infty$, $y \to \infty$
as $x \to \infty$, $y \to \infty$

c) $f(x) = -1x^6 + x^5 - x - 5$

lead. coeff.

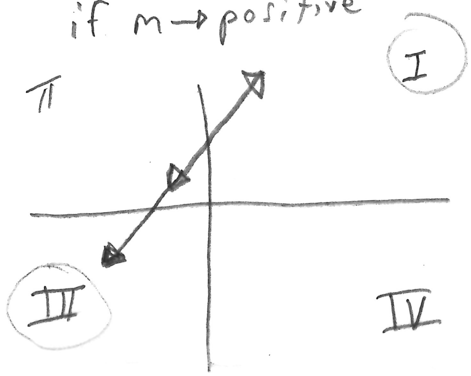
Degree	6
Even or Odd Degree	Even
Sign of Leading Coefficient	Negative
Max # of Turning Points	5
Max # of x-ints	6
End Behaviours	

as $x \to -\infty$, $y \to -\infty$
as $x \to \infty$, $y \to -\infty$

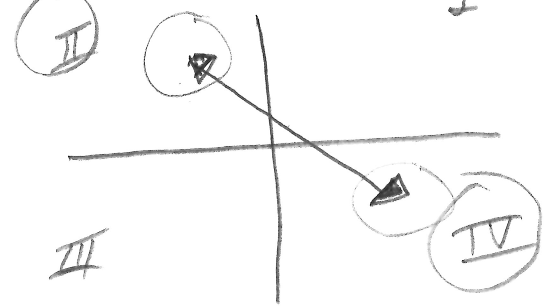
$y = mx + b$ degree = 1
(odd)

leading coefficient

if $m \rightarrow$ positive



if $m \rightarrow$ negative

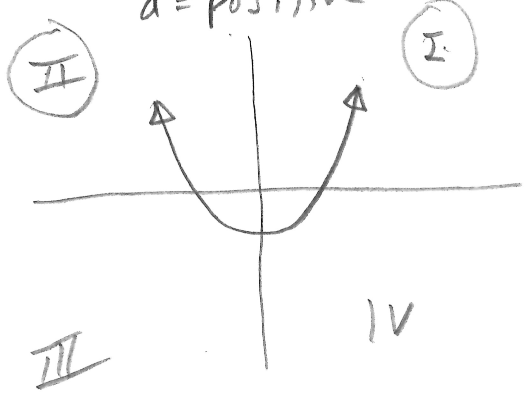


$y = ax^2 + bx + c$

leading coefficient

a = negative a = positive

a = positive



a = negative