Applications of Logarithmic Functions

Logarithmic functions are used to describe scenarios where an exponential change in one quantity corresponds with a linear change in another. Some examples of real-world applications of logarithmic functions include:

- The Richter scale for measuring earthquake intensities.
- The Decibel scale to represent the loudness of sounds.
- The pH scale used in chemistry to measure the acidity of a solution.

Example 1

The amplitude detected from the vibrations of an earthquake increases by a factor of 10 for each increase of 1 on the Richter scale. In other words, an earthquake that measures 7 on the Richter scale has an amplitude that is 10 times larger than one with a measure of 6 on the Richter scale. An earthquake with a reading of 8 on the Richter scale has an amplitude that is 100 times larger than one with a measure of 6 on the Richter scale.

$$\text{Richter Scale Magnitude} = \log(\text{Intensity})$$

measured in microns (10^{-6} m) of amplitude 100km from the epicenter

How many times more intense is an earthquake with magnitude of 6.8 on the Richter scale compared to one that has a measure of 4.3?

$RS_1 = \log(\text{Intensity}_1)$

$6.8 = \log(\text{Intensity}_1)$

$\text{Intensity}_1 = 10^{6.8}$

$RS_2 = \log(\text{Intensity}_2)$

$4.3 = \log(\text{Intensity}_2)$

$\text{Intensity}_2 = 10^{4.3}$

$\text{Factor} = \dfrac{\text{Intensity}_1}{\text{Intensity}_2}$

$= \dfrac{10^{6.8}}{10^{4.3}}$

$= 10^{2.5}$

$\cong 316$ times more intense

Example 2

The loudness of sound is measured in Decibels. For every linear increase of 10 Decibels, the sound intensity increases by a factor of 10. For example, sound that has a loudness of 90 dB is 100 times more intense than sound that has a loudness of 70 dB.

$$\text{Loudness(in dB)} = 10\log\left(\frac{I}{I_0}\right) \qquad \text{where } I_0 = 10^{-12}\text{W/m}^2$$

How many times more intense is the sound of a rock concert (at 120 dB) compared to normal conversation (at 60 dB)?

$L_{RC} = 10\log\left(\dfrac{I_{RC}}{I_0}\right)$

$\dfrac{120}{10} = \dfrac{\cancel{10}\log\left(\frac{I_{RC}}{10^{-12}}\right)}{\cancel{10}}$

$12 = \log\left(\dfrac{I_{RC}}{10^{-12}}\right)$

$\dfrac{I_{RC}}{10^{-12}} = \dfrac{10^{12}}{1}$

$I_{RC} = 10^{0}\ \text{W/m}^2$

$L_{NC} = 10\log\left(\dfrac{I_{NC}}{I_0}\right)$

$\dfrac{60}{10} = \dfrac{\cancel{10}\log\left(\frac{I_{NC}}{10^{-12}}\right)}{\cancel{10}}$

$6 = \log\left(\dfrac{I_{NC}}{10^{-12}}\right)$

$\dfrac{I_{NC}}{10^{-12}} = \dfrac{10^{6}}{1}$

$I_{NC} = 10^{-6}\ \text{W/m}^2$

$\text{Factor} = \dfrac{I_{RC}}{I_{NC}}$

$= \dfrac{10^0}{10^{-6}}$

$= 10^6$

$\therefore$ A million times more intense.

Example 3

The pH of a solution is a measure of the inverse hydrogen ion $[H^+]$ concentration in units of mol/L. As the pH increases, the amount of hydrogen ions in the solution decreases exponentially. For example, a solution with a pH of 6 is 100 times more acidic than a solution with a pH of 8. Pure distilled water has a pH of 7.

$$pH = -\log[H^+] \qquad \text{where } [H^+] \text{ is the concentration of } H^+$$

a) Determine the pH of hydrochloric acid that is secreted by the stomach lining and has a concentration of 0.03 mol/L.

$pH = -\log[H^+]$

$pH = -\log(0.03)$

$pH = 1.5$

b) Apple juice has a pH of 3. Determine the concentration of hydrogen ions in this solution.

$pH = -\log[H^+]$

$\frac{3}{-1} = \frac{-\log[H^+]}{-1}$

$-3 = \log[H^+]$

$[H^+] = 10^{-3}$

$[H^+] = 0.001 \text{ mol/L}$

c) If vinegar is 200 times more acidic than tomatoes which has a pH of 4.5, what is the pH of vinegar?

① $pH_t = -\log[H^+]_t$

$\frac{4.5}{-1} = \frac{-\log[H^+]_t}{-1}$

$-4.5 = \log[H^+]_t$

$[H^+]_t = 10^{-4.5} \text{ mol/L}$

② $[H^+]_v = 200 \times [H^+]_t$

$[H^+]_v = 200 \times 10^{-4.5} \text{ mol/L}$

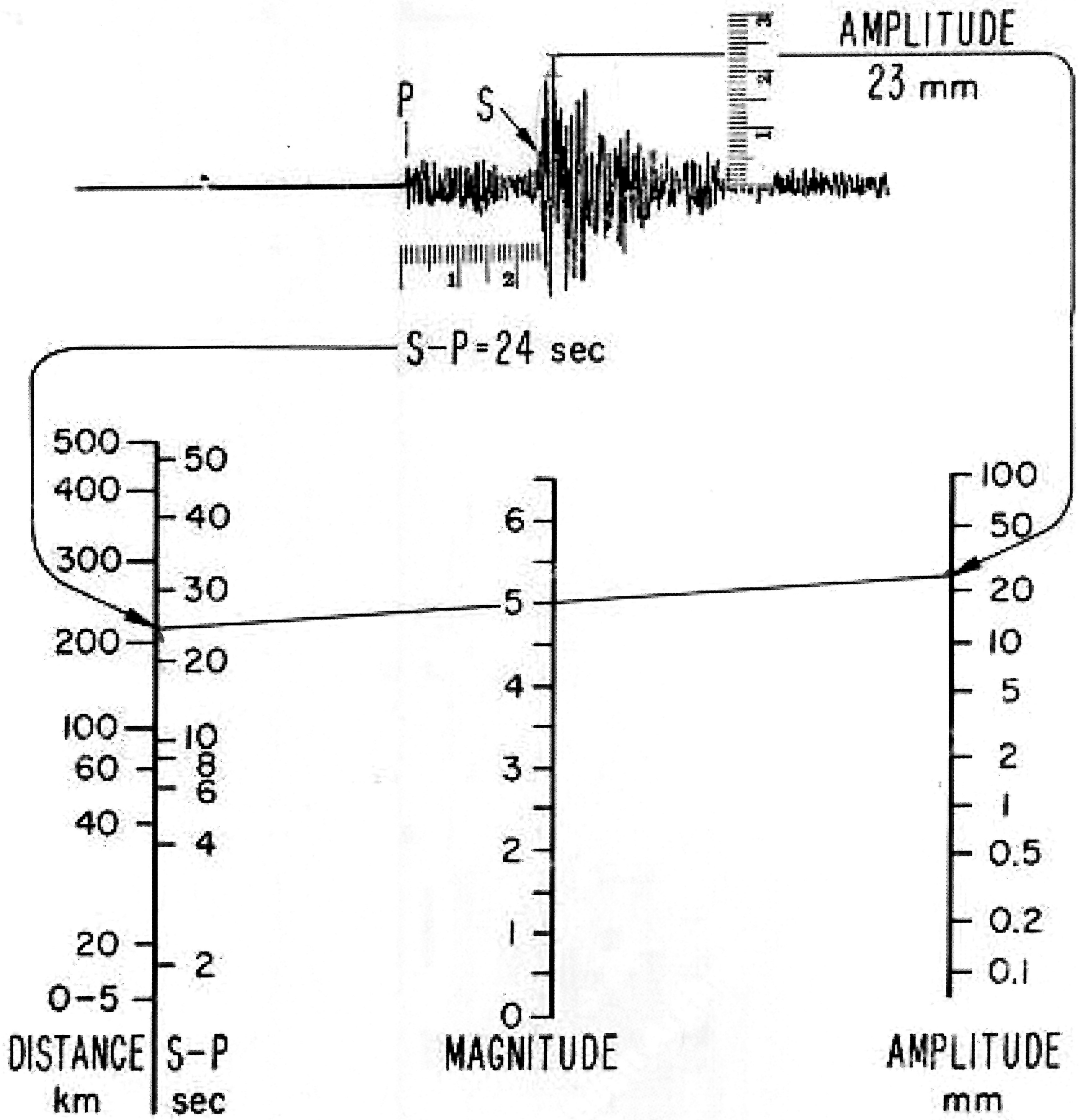
③ $pH_v = -\log[H^+]_v$

$pH_v = -\log(200 \times 10^{-4.5})$

$pH_v \doteq 2.2$

Homework: pg 499 # 1, 2, 3, 4, 5ac, 6ac, 7, 8, 10, 13, 14, 15, 17

THE RICHTER SCALE



TO DETERMINE THE MAGNITUDE OF AN EARTHQUAKE WE CONNECT ON THE CHART

A. THE MAXIMUM AMPLITUDE RECORDED BY A STANDARD SEISMOMETER, AND
B. THE DISTANCE OF THAT SEISMOMETER FROM THE EPICENTER OF THE EARTHQUAKE (OR THE DIFFERENCE IN TIMES OF ARRIVAL OF THE P AND S WAVES)

BY A STRAIGHT LINE, WHICH CROSSES THE CENTER SCALE AT THE MAGNITUDE