Solving Equations with Fractions

There are multiple techniques to consider to solve an equation that has fractions:

- Cross Multiplication
- Common Denominator
- Multiplying Through

1. Cross Multiplication

$\frac{\quad}{\quad} = \frac{\quad}{\quad}$

This technique is used when there is a **single fraction** on **both sides** of the equal sign.

Example: Solve for x.

a) $\frac{3}{x} = \frac{9}{12}$

$\frac{9x}{9} = \frac{36}{9}$

$x = 4$

b) $\frac{10}{x} = \frac{-2}{1}$

$\frac{-2x}{-2} = \frac{10}{-2}$

$x = -5$

c) $\frac{2}{x+1} = \frac{1}{2}$

$1(x+1) = 4$

$x+1 = 4$

$x = 4-1$

$x = 3$

2. Common Denominator

A common denominator can be used when there are **multiple fractions** on one or both sides **of the equal sign**.

Example: Solve for x.

a) $\frac{x}{2} + \frac{x}{5} = 14$

$\frac{5x}{10} + \frac{2x}{10} = 14$

$\frac{7x}{10} = \frac{14}{1}$

$\frac{7x}{7} = \frac{140}{7}$

$x = 20$

b) $\frac{2x}{3} - \frac{x}{5} = \frac{7}{15}$

$\frac{10x}{15} - \frac{3x}{15} = \frac{7}{15}$

$\frac{7x}{15} = \frac{7}{15}$

$\frac{105x}{105} = \frac{105}{105}$

$x = 1$

c) $\frac{x+3}{2} - \frac{2}{1} = \frac{x}{3}$

$\frac{(x+3)}{2} - \frac{4}{2} = \frac{x}{3}$

$\frac{(x+3)-4}{2} = \frac{x}{3}$

$\frac{x-1}{2} = \frac{x}{3}$

$3(x-1) = 2x$

$3x-3 = 2x$

$3x-2x = 3$

$x = 3$

3. Multiplying Through

To multiply through, we again determine a common denominator, but this time, we multiply each term by this value cancelling out all fractions.

Example: Solve for x.

a) $\frac{x}{2} + \frac{x}{5} = \frac{14}{1}$

$5x + 2x = 140$

$\frac{7x}{7} = \frac{140}{7}$

$x = 20$

b) $\frac{2x}{3} - \frac{x}{5} = \frac{7}{15}$

$10x - 3x = 7$

$\frac{7x}{7} = \frac{7}{7}$

$x = 1$

c) $\frac{x+3}{2} - \frac{2}{1} = \frac{x}{3}$

$3(x+3) - 12 = 2x$

$3x + 9 - 12 = 2x$

$3x - 3 = 2x$

$3x - 2x = 3$

$x = 3$

Practice Questions

Solve each question using a different technique.

a) $\frac{x}{4} + \frac{x+1}{3} = \frac{5}{1}$

$3x + 4(x+1) = 60$

$3x + 4x + 4 = 60$

$7x = 60 - 4$

$\frac{7x}{7} = \frac{56}{7}$

$x = 8$

b) $\frac{4}{x} = \frac{6}{x+1}$

$6x = 4(x+1)$

$6x = 4x + 4$

$6x - 4x = 4$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

c) $\frac{x}{3} - \frac{5}{1} = \frac{2x-18}{2}$

$\frac{x}{3} - \frac{15}{3} = \frac{2x-18}{2}$

$\frac{x-15}{3} = \frac{2x-18}{2}$

$3(2x-18) = 2(x-15)$

$6x - 54 = 2x - 30$

$6x - 2x = -30 + 54$

$\frac{4x}{4} = \frac{24}{4}$

$x = 6$