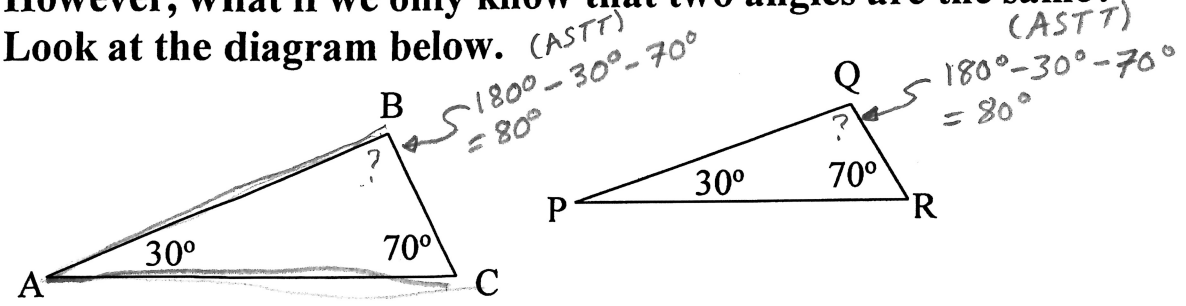
Similar Triangles: Part 2

It has been shown that, if the three angles in one triangle are equal **to the three angles in another triangle then the two triangles are** similar **.**

However, what if we only know that two angles are the same? Look at the diagram below. (ASTT)



Are these triangles similar? Yes!

We know that <BAC = $\angle QPR$ **(30°)**
<BCA = $\angle QRP$ **(70°)**

But what about <ABC and <PQR?

Using the rule ASTT, we find that both < B and < Q are equal to 80° **.**

This example shows that if a pair of angles in one triangle are equal to a corresponding pair of angles in another triangle then the third set of corresponding angles must also be equal **.**

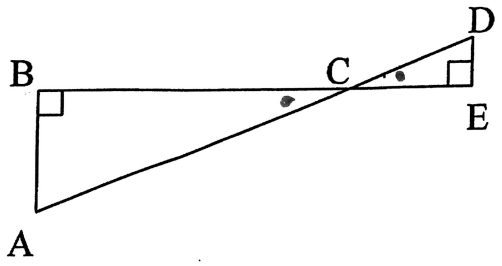
Therefore, we no longer need to show that all three angles in two triangles are equal to prove that the triangles are similar.

Angle-Angle-Similarity Rule (AA~)

If two angles in one triangle are the same as two angles in another triangle, then the two triangles are similar by AA~.

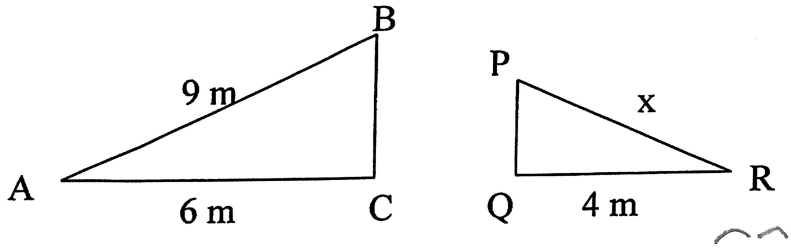
Examples

1. Prove that two triangles are similar.



Proof
Since $\angle ABC = \angle DEC$ (⊾)
$\angle BCA = \angle DCE$ (OAT)
Then $\triangle ABC \sim \triangle DEC$ (AA~)

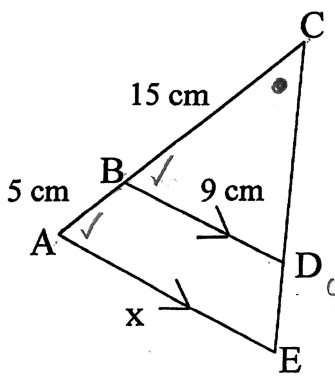
2. Determine the length of x.



Given: ΔABC~ ΔRPQ

Solution
Since $\triangle ABC \sim \triangle RPQ$
then $AB:BC:CA = RP:PQ:QR$
$9:BC:6 = x:PQ:4$
$\frac{9}{x} = \frac{BC}{PQ} = \frac{6}{4}$
$\frac{9}{x} = \frac{6}{4}$
$\frac{6x}{6} = \frac{36}{6}$
$x = 6m$

3. Determine the length of x.

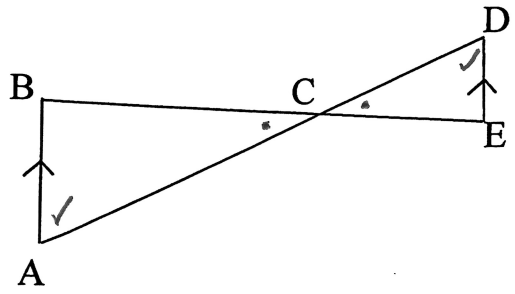


Proof
Since $\angle CBD = \angle CAE$ (PLT-F)
$\angle BCD = \angle ACE$ (common)
then $\triangle BCD \sim \triangle ACE$ (AA~)

Solution
Since $\triangle BCD \sim \triangle ACE$
then $BC:CD:DB = AC:CE:EA$
$15:CD:9 = 20:CE:x$
$\frac{15}{20} = \frac{CD}{CE} = \frac{9}{x}$
$\frac{15}{20} = \frac{9}{x}$
$\frac{15x}{15} = \frac{180}{15}$
$x = 12$ cm

Practice

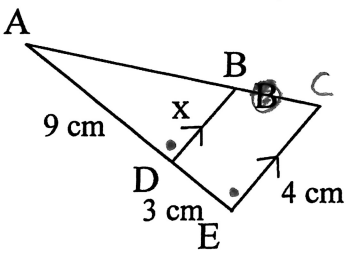
1. Prove that two triangles are similar.



Since $\angle BCA = \angle DCE$ (OAT)
$\angle BAC = \angle EDC$ (PLT-Z)
then $\triangle ABC \sim \triangle DEC$ (AA~)

2. Determine the length of the side marked x; be sure to include a proof.

a)



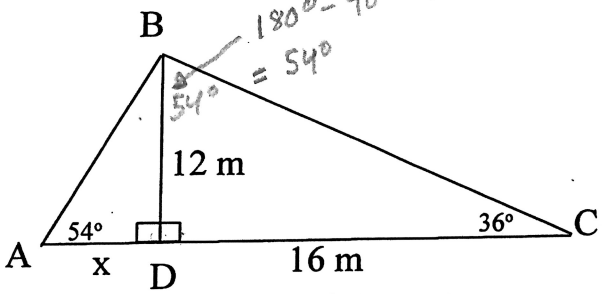
Since $\angle DAB = \angle EAC$ (common)
$\angle ADB = \angle AEC$ (PLT-F)
then $\triangle ABD \sim \triangle ACE$ (AA~)
Since $\triangle ABD \sim \triangle ACE$
$AB:BD:DA = AC:CE:EA$
$AB:x:9 = AC:4:12$

$\frac{AB}{AC} = \frac{x}{4} = \frac{9}{12}$

$\frac{12x}{12} = \frac{36}{12}$

$x = 3$ cm

b)



Since $\angle ADB = \angle CDB$ (90°)
$\angle BAD = \angle CBD$ (54°)
then $\triangle ABD \sim \triangle BCD$
Since $\triangle ABD \sim \triangle BCD$
then $AB:BD:DA = BC:CD:DB$
$AB:12:x = BC:16:12$

$\frac{AB}{BC} = \frac{12}{16} = \frac{x}{12}$

$\frac{16x}{16} = \frac{144}{16}$

$x = 9$ m

Answers: 1. $\Delta ABC \sim \Delta DEC$, 2. a) 3 cm b) 9 m