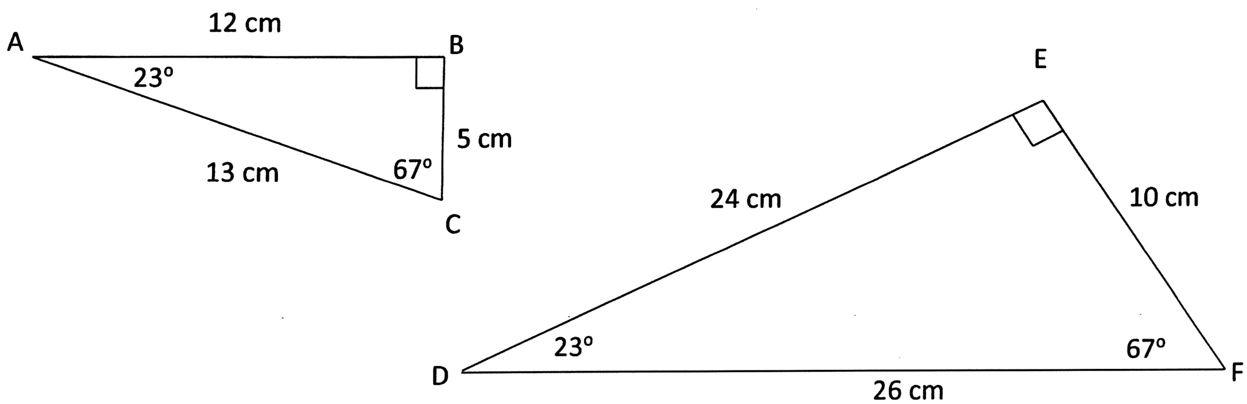
Similar Triangles – Introduction

In general, two triangles are considered to be similar if one triangle is a magnified (enlarged) version of the other.

For example, consider the following two triangles:



From these two triangles, we can see that ΔDEF is an enlarged copy of ΔABC; therefore, the two triangles are similar. Mathematically, this statement of similarity can be written $\triangle DEF \sim \triangle ABC$. Note that the order for which we write the letters is VERY important.

Quantitatively, two triangles are considered to be similar if either of the following two statements are true:

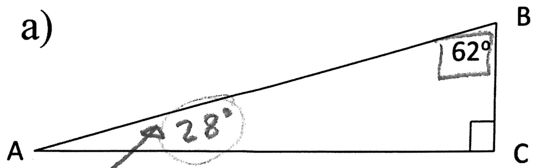
1. The lengths of the three sides of one triangle are proportional to the lengths of the corresponding three sides in the other triangle. For the example above, we may write $DE:EF:FD = AB:BC:CA$.
   $24:10:26 = 12:5:13$
2. The three angles in one triangle are equal to the corresponding three angles in the other triangle. For the example above,
   - $\angle A = \angle D$ (23°)
   - $\angle B = \angle E$ (∟)
   - $\angle C = \angle F$ (67°)

It is important to note that, if one of the above statements is true, then the triangles are similar and the other statement must also be true.

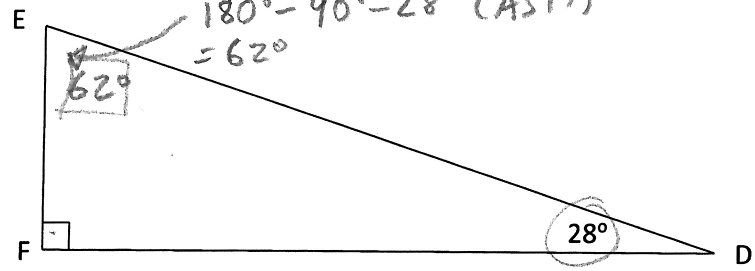
Examples

Prove that the following pairs of triangles are similar.

a)



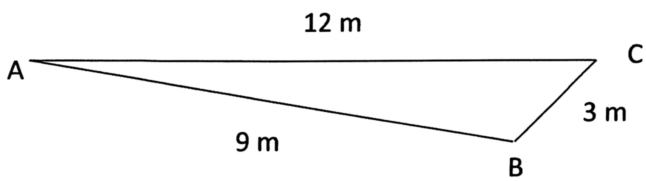
180° − 90° − 62° (ASTT)
= 28°

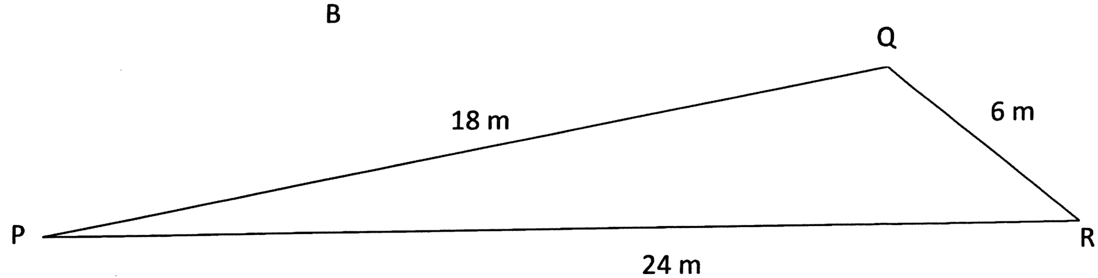


180° − 90° − 28° (ASTT)
= 62°

Since ∠BAC = ∠EDF (28°)
∠ABC = ∠DEF (62°)
∠ACB = ∠DFE (90°)
then △ABC ~ △DEF (AAA~)

b)

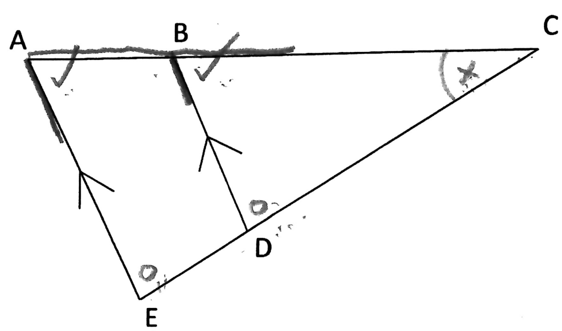




Since $\frac{AC}{PR} = \frac{12}{24} = \frac{1}{2}$, $\frac{BC}{QR} = \frac{3}{6} = \frac{1}{2}$, $\frac{AB}{PQ} = \frac{9}{18} = \frac{1}{2}$

△ABC ~ △PQR (SSS~)

c)

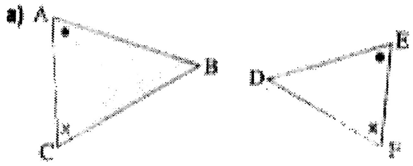


Since ∠CAE = ∠CBD (PLT-F)
∠AEC = ∠BDC (PLT-F)
∠ACE = ∠BCD (common)
Then △BCD ~ △ACE (AAA~)

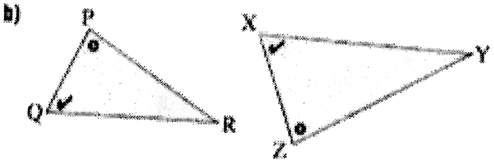
Introduction to Similar Triangles – Practice

1. For each pair of similar triangles, list the corresponding sides and angles.

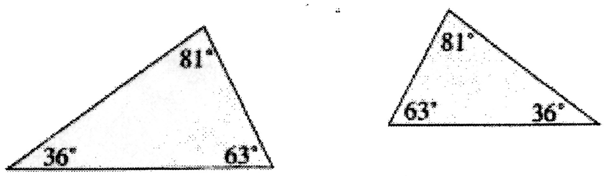
a)



b)



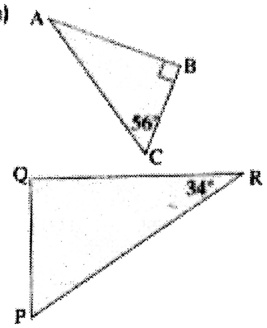
2. These triangles have the same shape, but different orientations.



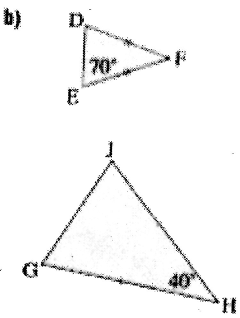
Are the triangles similar? Explain.

3. The triangles in each pair are similar. In each case, state the measures of the angles that are not marked.

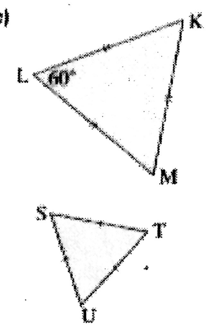
a)



b)

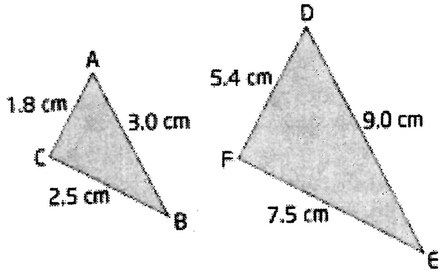


c)

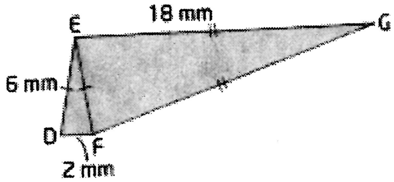


4. Name a pair of similar triangles in each diagram and explain why they are similar.

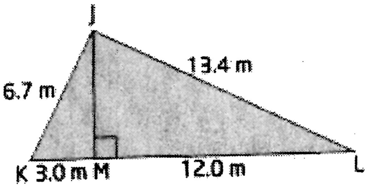
a)



b)

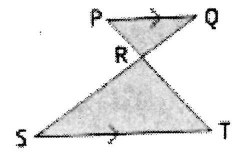


c)

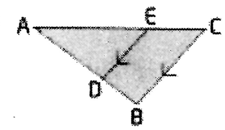


5. Name a pair of similar triangles in each diagram and explain why they are similar.

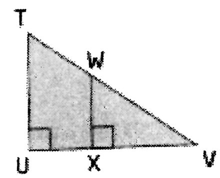
a)



b)



c)



Answers

1. a) ∠A = ∠E, ∠B = ∠D, ∠C = ∠F; AB corresponds to ED; BC corresponds to DF; CA corresponds to FE.
   b) ∠P = ∠Z, ∠Q = ∠X, ∠Y = ∠R; PQ corresponds to ZX; QR corresponds to XY; RP corresponds to YZ.

2. Yes

3. a) ∠A = 34°, ∠P = 56°, ∠Q = 90°
   b) ∠D = 70°, ∠F = 40°, ∠J = 70°, ∠G = 70°
   c) ∠K = 60°, ∠M = 60°, ∠S = 60°, ∠T = 60°, ∠U = 60°

4. a) △ABC ~ △DEF; ratios of corresponding sides are all equal to $\frac{1}{3}$.
   b) △DEF ~ △EGF; ratios of corresponding sides are all equal to $\frac{1}{3}$.
   c) △JKM ~ △LJM; ratios of corresponding sides are all equal to $\frac{1}{2}$.

5. a) △PQR ~ △TSR; ∠P = ∠T and ∠Q = ∠S because they are alternate angles. Also, ∠PRQ = ∠TRS because they are opposite angles.
   b) △ABC ~ △ADE; ∠A is common to both triangles; ∠B = ∠D and ∠C = ∠E because they are corresponding angles of parallel lines.
   c) △TUV ~ △WXV; ∠V is common to both triangles, ∠U = ∠X because they are both right angles. Also, ∠T = ∠W because they are corresponding angles of parallel lines.