

Using Algebra in Problem Solving

For some problems, it is helpful to use algebra.
 Problem solving often involves writing word expressions as algebraic expressions.

Write an algebraic expression for each of the following:

Word Expression	Algebraic Expression
1. A number which is four more than b.	$b + 4$
2. A number which is four less than x.	$x - 4$
3. A number which is two times y.	$2y$
4. Five less than twice w.	$2w - 5$
5. A number which is x more than ten.	$10 + x$
6. Four more than six times a.	$6a + 4$
7. Two less than four times w.	$4w - 2$
8. Eight more than $(x + 2)$.	$(x + 2) + 8 = x + 10$

Rebecca is now n years old. Write an expression for:

- 10
45
- | | |
|--|-----------------------------------|
| a) Her age five years from now. | $n + 5$ |
| b) Her age ten years ago. | $n - 10$ |
| c) The age of someone twice as old as her. | $2n$ |
| d) Someone one half of her age. | $\frac{n}{2}, \frac{1}{2}n, 0.5n$ |
| e) Someone three more than twice her age. | $2n + 3$ |

A Guelph high school has an enrolment of a students. Write an algebraic expression for the enrolment of:

- | | |
|---|---------------------|
| a) A school having 150 more students. | $a + 150$ |
| b) A school having 3 times as many students. | $3a$ |
| c) A school having 30 more than twice as many students. | $2a + 30$ |
| d) A school having 75 less than half as many students. | $\frac{1}{2}a - 75$ |

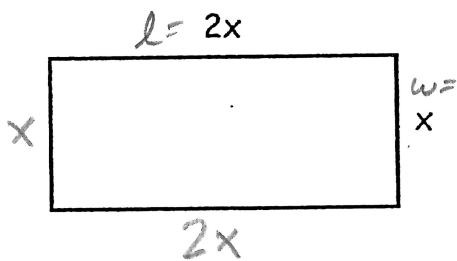
If y represents an integer, write an algebraic expression for:

- | | |
|---|--------------------------------------|
| a) Four less than the integer. | <u>$y - 4$</u> |
| b) Five more than the integer. | <u>$y + 5$</u> |
| c) One third of the integer. | <u>$\frac{1}{3}y$</u> |
| d) The integer decreased by seven. | <u>$y - 7$</u> |
| e) Four less than twice the integer. | <u>$2y - 4$</u> |
| f) One more than three quarters of the integer. | <u>$\frac{3}{4}y + 1$</u> |

State an algebraic expression for:

- | | |
|---|-------------------------------|
| a) The value in cents of a nickels. | <u>$5a$</u> |
| b) The value in cents of x dimes. | <u>$10x$</u> |
| c) The value in cents of y quarters. | <u>$25y$</u> |
| d) The value in nickels ^{cents} of z dollars. | <u>$100z$</u> |
| e) The value in cents of x dimes <u>and</u> y quarters. | <u>$10x + 25y$</u> |
- +

Find an expression for the perimeter and area of the following rectangle:

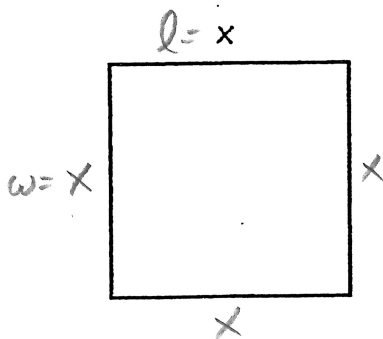


$$\begin{aligned} \text{Perimeter} &= 2(l + w) \\ &= 2(2x + x) \\ &= 2(3x) \\ &= 6x \end{aligned}$$

$$\begin{aligned} \text{Area} &= lw \\ &= 2x(x) \\ &= 2x^2 \end{aligned}$$

$$* \begin{cases} x + x = 2x \\ x(x) = x^2 \end{cases}$$

Find an expression for the perimeter and area of the square below:



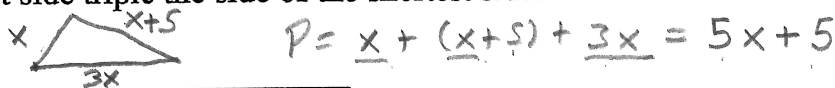
$$\begin{aligned} \text{Perimeter} &= x + x + x + x \\ &= 4x \end{aligned}$$

$$\begin{aligned} \text{Area} &= lw \\ &= (x)(x) \\ &= x^2 \end{aligned}$$

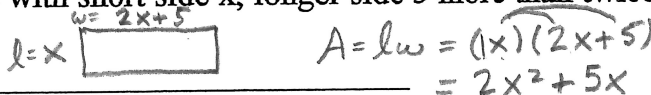
1. Translate each of the following phrases into a mathematical expression.
In each case let x represent the number

- a) four more than the number $x+4$ b) four times the number $4x$
- c) the next consecutive number to x $x+1$ d) the value of x dimes $10x$
- e) You have 10 coins in dimes or quarters.
If you have x dimes then you must have $10-x$ quarters.
$$\begin{array}{r} 10-3=7 \\ d \quad q \end{array}$$

f) The perimeter of a triangle with shortest side x , next largest side five more than the shortest side and largest side triple the side of the shortest side.



g) The area of a rectangle with short side x , longer side 5 more than twice the shorter side.



Complete # 2 and 3 on a separate sheet

2. Solve each of the following problems using the 5 step algorithm for Problem Solving

- a) Find three consecutive even numbers whose sum is 48.
- b) You have in your pocket 23 coins either dimes or quarters. ^{add} The value of the coins is \$5.00. Find the number of dimes in your pocket.
- c) In the cafeteria a hamburger costs 75 cents more than a pop. For lunch you buy two hamburgers and a pop. The total cost is \$4.80. Find the cost of a pop at the school cafeteria.
- d) A triangle has three sides such that the largest side is four times the length the middle side, And the middle side is 5 cm longer than the shortest side. If the perimeter of the triangle is 73 cm find the length of the shortest side.

3. Solve each of the following problems using the formulae given in each question:

- a) The cost to produce a year book is given by the formula $C = 12n + 400$, where n is the number of year books produced and C is the cost. Using the formula find the cost of producing 500 year books. If you have raised \$7840, how many year books could be produced?
- b) The formula for converting temperatures between Celsius degrees and Fahrenheit degrees is given by the rule $F = 1.8C + 32$, where F is the temperature in Fahrenheit and C is the temperature in Celsius. a) If the temperature was 10 degrees Celsius, find the temperature in Fahrenheit. If the temperature was 80 degrees Fahrenheit, find the temperature in Celsius.

2a) Let x represent the first number.
 Let $x+2$ represent the second number.
 Let $x+4$ represent the third number.

$$\text{1st\#} + \text{2nd\#} + \text{3rd\#} = \text{sum}$$

$$(x) + (x+2) + (x+4) = 48$$

$$3x + 6 = 48$$

$$3x = 48 - 6$$

$$\frac{3x}{3} = \frac{42}{3}$$

$$x = 14$$

The numbers are 14, 16, 18

2b) Let d represent the number of dimes.
 Let $23-d$ represent the number of quarters.

$$(\text{\# of dimes})(0.10) + (\text{\# of quarters})(0.25) = \text{total}$$

$$0.10d + 0.25(23-d) = 5.00$$

$$0.10d + 5.75 - 0.25d = 5.00$$

$$-0.15d = 5.00 - 5.75$$

$$-0.15d = -0.75$$

$$-0.15 \quad -0.15$$

$$d = 5$$

\therefore There are 5 dimes
 and 18 quarters.

2c) Let p represent the cost of one pop (\$).
 Let $p+0.75$ represent the cost of one hamburger (\$).

$$2 \times (\text{cost of a hamburger}) + 1 \times (\text{cost of a pop}) = \text{total}$$

$$2(p+0.75) + 1(p) = 4.80$$

$$2p + 1.50 + 1p = 4.80$$

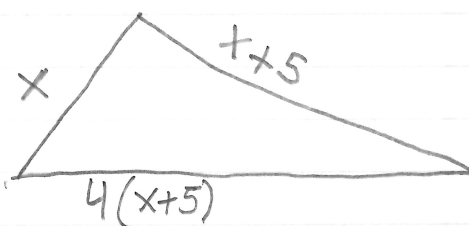
$$3p = 4.80 - 1.50$$

$$\frac{3p}{3} = \frac{3.30}{3}$$

$$p = \$1.10$$

\therefore The cost of one pop is \$1.10.
 and a hamburger costs \$1.85.

2d)



large \rightarrow medium \rightarrow small

$$\text{side}_1 + \text{side}_2 + \text{side}_3 = \text{perimeter}$$

$$(x) + (x+5) + 4(x+5) = 73$$

$$x + x + 5 + 4x + 20 = 73$$

$$x + x + 4x = 73 - 5 - 20$$

$$6x = 48$$

$$\frac{6x}{6} = \frac{48}{6}$$

$$x = 8 \text{ cm}$$

\therefore The side lengths are 8cm, 13cm, and 52cm

$$3b) \text{ i) } F = 1.8C + 32$$

$$\text{set } C = 10$$

$$F = 1.8(10) + 32$$

$$F = 18 + 32$$

$$F = 50$$

\therefore It is 50°F

$$\text{ii) } F = 1.8C + 32$$

$$\text{set } F = 80$$

$$80 = 1.8C + 32$$

$$-1.8C = 32 - 80$$

$$\underline{-1.8C = -48}$$

$$\underline{-1.8} \quad \underline{-1.8}$$

$$C = 26.7$$

\therefore It is 26.7°C

Homework # 3a

Word Problem Practice

1. The sum of four consecutive numbers is 118. What are the numbers?

(Answer: 28, 29, 30, 31)

2. You have \$2.45 in your pocket which is made up of 17 coins; all are quarters and dimes. How many quarters and dimes do you have?

(Answer: 5 quarters, 12 dimes)

3. Albert goes shopping and purchases 3 footballs and 2 soccer ball and 1 pair of shoes for for \$190. Footballs cost \$5 more than soccer balls. One pair of shoes cost twice as much as one soccer ball. How much does one soccer ball cost?

(Answer: Soccer ball \$25, Football = \$30, Shoes = \$50)