Tan Inverse

Consider the equations $x^2 = 16$, $2x = 16$ and $x + 2 = 16$.
If we want to isolate x in each of these equations, we do the following

$x^2 = 16$

$\sqrt{x^2} = \sqrt{16}$

$x = 4$

and

$2x = 16$

$\frac{2x}{2} = \frac{16}{2}$

$x = 8$

and

$x + 2 = 16$

$x + 2 - 2 = 16 - 2$

$x = 14$

$x + 2 = 16$

$x = 16 - 2$

$x = 14$

We use opposite (or inverse) operations to isolate x.

Opposite (or inverse) operations can also be used to isolate Θ when it is bound by a trigonometric function.

Example 1

Suppose $\tan\theta = 1$. Can we solve for Θ? Yes!

tan inverse

$\tan\theta = 1$ ← shift, tan (2nd)

$\tan^{-1}(\tan\theta) = \tan^{-1}(1)$

$\theta = 45^\circ$ ← angle

Notice that when we use inverse trigonometric functions, the answer is an angle.

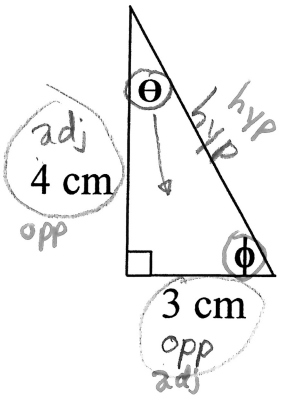
Typically, we would skip the second step above and write the solution as follows:

$$\tan\theta = 1$$
$$\theta = \tan^{-1}(1)$$
$$\theta = 45^\circ$$

Remember to keep your calculator in 'degree' mode.

Example 2

Use $\tan^{-1}$ to find the enclosed angles



$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$\theta \cong 36.9°$

$\tan\phi = \frac{opp}{adj}$

$\tan\phi = \frac{4}{3}$

$\phi = \tan^{-1}\left(\frac{4}{3}\right)$

$\phi \cong 53.1°$

$4 \div 3 =$, 2nd, tan

Note: The inverse functions can be activated on your calculator by first pressing the 2nd/shift button followed by the 'tan' button.

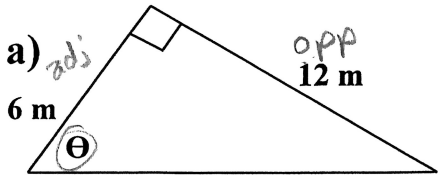
Notice, in the above example, that all of the angles in the triangle add up to 180° as they should by ASTT.

Example 3

SOH CAH TOA

Determine the angle θ in each diagram.

a)



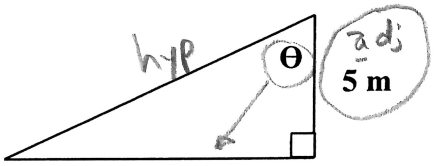
$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{12}{6}$

$\theta = \tan^{-1}(2)$

$\theta \cong 63.4°$

b)



$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{14}{5}$

$\theta \cong \tan^{-1}\left(\frac{14}{5}\right)$

$\theta \cong 70.3°$

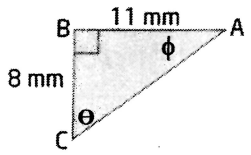
Homework – Inverse Tangent

1. Find the measure of each angle, to the nearest degree.

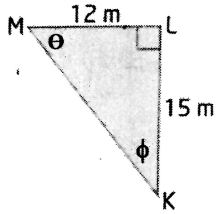
a) $\tan\theta = 1.5$ **b)** $\tan\theta = \frac{6}{7}$ **c)** $\tan\theta = \frac{15}{9}$

2. Find the measures of both acute angles in each triangle, to the nearest degree.

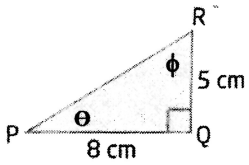
a)



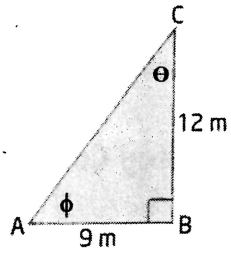
b)



c)



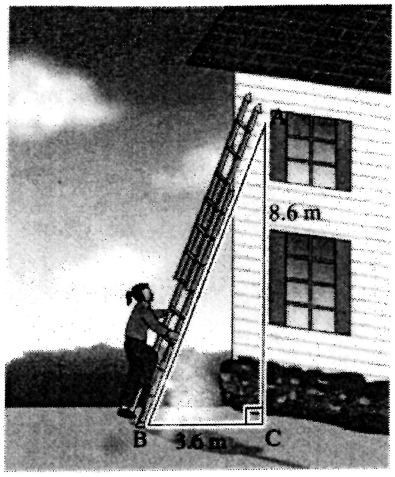
d)



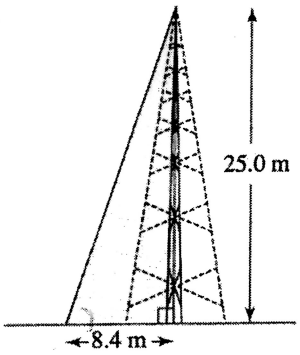
3. An extension ladder is leaning against a wall. The foot of the ladder is 3.6 m from the wall. The top of the ladder reaches 8.6 m up the wall.

a) Calculate the measure of the angle formed by the ladder and the ground.

b) Calculate the length of the ladder.



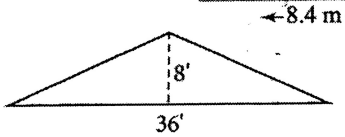
4. A guy wire supports a tower. The wire is attached to the tower at a height of 25.0 m. The guy wire is attached to the ground 8.4 m from the base of the tower. Calculate the measure of the angle formed by the guy wire and the ground.



5. A roof truss spans a width of 36' and has a height of 8'. The sides that join at the top of the truss are equal in length.

a) Find the angle formed by the two sides that join at the top of the truss.

b) Find the length of the equal sides.



Answers:

1. a) 56° b) 41° c) 59°

2. a) Θ = 54°, ϕ = 36°

b) Θ = 51°, ϕ = 39°

c) Θ = 32°, ϕ = 58°

d) Θ = 37°, ϕ = 53°

3. a) 67° b) 9.3 m

4. 71° 5. a) 132° b) 19.7 feet