

Tangent's Friend the Sine and Cosine Ratio

Thus far, the only trigonometric equation that we have been using is the *tangent ratio*. This equation is limited since it only makes use of the opposite and adjacent; the hypotenuse is not part of the tangent ratio.

There are two other trigonometric ratios that can be used,

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

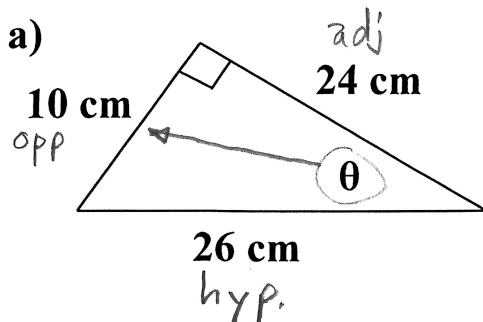
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

All three ratios can be memorized using the following acronym:

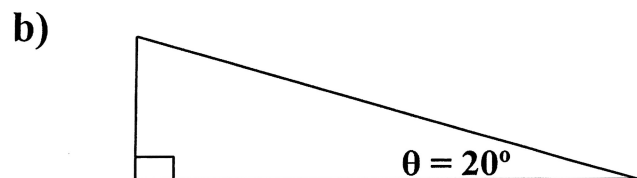
SOH CAH TOA

Example 1:

For the following triangles, label the sides opposite, adjacent and hypotenuse. Calculate $\sin \theta$, $\cos \theta$, and $\tan \theta$.



$$\begin{aligned}\sin \theta &= \frac{\text{opp}}{\text{hyp}} & \cos \theta &= \frac{\text{adj}}{\text{hyp}} & \tan \theta &= \frac{\text{opp}}{\text{adj}} \\ &= \frac{10 \text{ cm}}{26 \text{ cm}} & &= \frac{24}{26} & &= \frac{10}{24} \\ &= \frac{5}{13} & &= \frac{12}{13} & &= \frac{5}{12}\end{aligned}$$

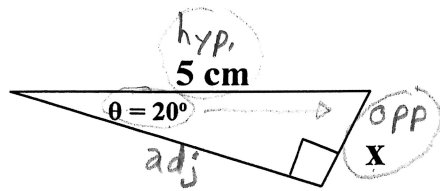


$$\begin{aligned}\sin \theta &= \sin(20^\circ) & \cos \theta &= \cos(20^\circ) & \tan \theta &= \tan(20^\circ) \\ &\approx 0.342 & &\approx 0.940 & &\approx 0.364\end{aligned}$$

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Example 2:

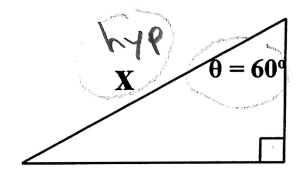
Determine the length by using the sine equation (to 1 decimal place).

a) 

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\frac{\sin(20^\circ)}{1} = \frac{x}{5}$$

$x = 5 \sin(20^\circ)$
 $x \approx 1.7 \text{ cm}$

b) 

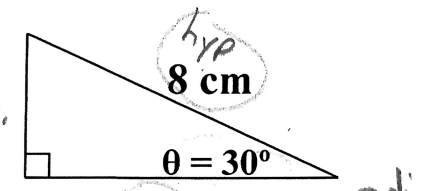
$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\frac{\sin(60^\circ)}{1} = \frac{8}{x}$$

$x \sin(60^\circ) = 8$
 $\frac{x \sin(60^\circ)}{\sin(60^\circ)} = \frac{8}{\sin(60^\circ)}$
 $x \approx 9.2 \text{ cm}$

Example 3:

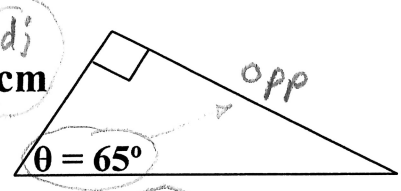
Determine the length x by using the cosine equation (to 1 decimal place).

a) 

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\frac{\cos(30^\circ)}{1} = \frac{x}{8}$$

$x = 8 \cos(30^\circ)$
 $x \approx 6.9 \text{ cm}$

b) 

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

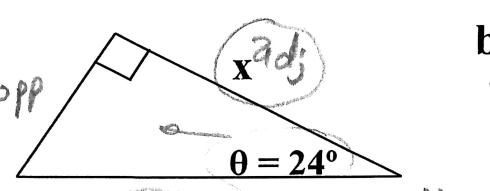
$$\frac{\cos(65^\circ)}{1} = \frac{2}{x}$$

$x \cos(65^\circ) = 2$
 $\frac{x \cos(65^\circ)}{\cos(65^\circ)} = \frac{2}{\cos(65^\circ)}$
 $x \approx 4.7 \text{ cm}$

Example 4:

Determine the length x using a trigonometric ratio.

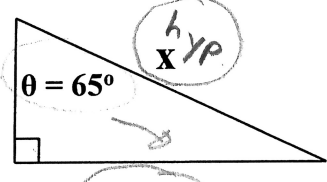
SOH CAH TOA

a) 

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\frac{\cos(24^\circ)}{1} = \frac{x}{14}$$

$x = 14 \cos(24^\circ)$
 $x \approx 12.8 \text{ m}$

b) 

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\frac{\sin(65^\circ)}{1} = \frac{5}{x}$$

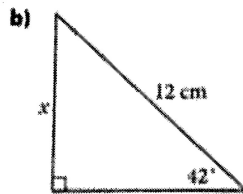
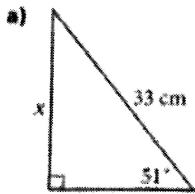
$x \sin(65^\circ) = 5$
 $\frac{x \sin(65^\circ)}{\sin(65^\circ)} = \frac{5}{\sin(65^\circ)}$
 $x \approx 5.5 \text{ km}$

Practice - Introduction to Sine and Cosine Ratio

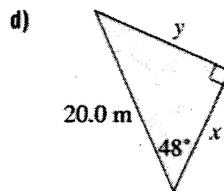
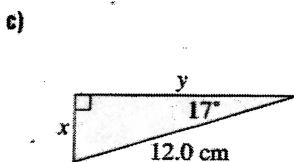
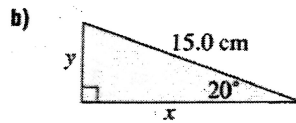
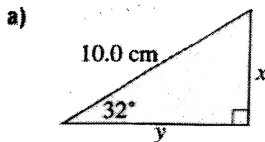
1. Use a scientific calculator to find each value to four decimal places.

- | | | |
|--------------------|--------------------|--------------------|
| a) $\sin 42^\circ$ | b) $\sin 33^\circ$ | c) $\cos 19^\circ$ |
| d) $\sin 88^\circ$ | e) $\cos 74^\circ$ | f) $\cos 38^\circ$ |
| g) $\sin 45^\circ$ | h) $\cos 42^\circ$ | |

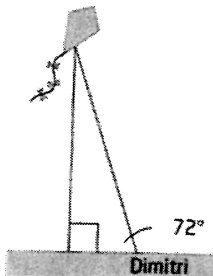
2. Find x to the nearest tenth of a centimetre.



3. Determine each value of x . Use the Pythagorean Theorem to determine each value of y .

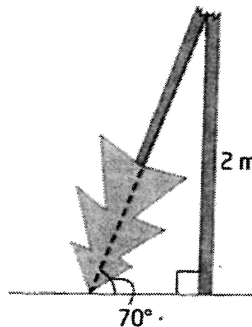


4. Dmitri has let out 40 m of his kite string, which makes an angle of 72° with the horizontal ground.



- Find the height of the kite, to the nearest metre.
- Suppose the Sun is shining directly above the kite. How far is the kite's shadow from Dimitri, to the nearest metre?

5. A tree is splintered by lightning 2 m up its trunk, so that the top part of the tree touches the ground. The angle the top of the tree forms with the ground is 70° . Before it was splintered, how tall was the tree, to the nearest tenth of a metre?



Answers: 1.a) 0.6691 b) 0.5446 c) 0.9455 d) 0.9994 e) 0.2756 f) 0.7880 g) 0.7071

h) 0.7431 2. a) 25.6 cm b) 8.0 cm 3. a) $x = 5.3$ cm, $y = 8.5$ cm b) $x = 14.1$ cm, $y = 5.1$ cm

c) $x = 3.5$ cm, $y = 11.5$ cm d) $x = 13.4$ m, $y = 14.9$ m 4. a) 38.0 m b) 12.4 m 5. 4.13 m