

Applications of Quadratic Functions: Part 2

Mr. Ryan sells a new text messaging unit that also completes homework. This device is called the iStudent. The total profit y depends on the selling price x and is modeled by the following equation:

$$y = -100(x - 50)(x - 150)$$

where

- y is the profit in dollars.
- x is the selling price of each unit.

a) How much profit will Mr. Ryan make if he sells the units for \$75?

$$\begin{aligned} \text{set } x &= 75 \\ y &= -100(75 - 50)(75 - 150) \\ y &= -100(25)(-75) \end{aligned} \rightarrow = \$187500$$

b) What are the x-intercepts?

$$x\text{-ints} = 50 \text{ \& } 150$$

c) Expand the equation to express it in standard form.

$$\begin{aligned} y &= -100(x - 50)(x - 150) \\ y &= -100(x^2 - 150x - 50x + 7500) \\ y &= -100(x^2 - 200x + 7500) \end{aligned} \rightarrow y = -100x^2 + 20000x - 750000$$

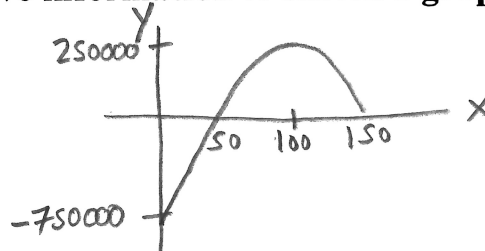
d) What is the y-intercept?

$$y\text{-int} = -750000$$

e) Determine the vertex of the above function.

$$\begin{aligned} \text{Vertex} \\ x &= \frac{50 + 150}{2} \\ x &= 100 \\ y &= -100(100 - 50)(100 - 150) \\ &= -100(50)(-50) \\ y &= 250000 \end{aligned} \rightarrow \therefore \text{The vertex is } (100, 250000)$$

f) Use the above information to sketch a graph of Profit vs. Price.



g) At what price will Mr. Ryan make the most profit and what will that profit be?

He can make a maximum profit of \$250000 if he sells the iStudent for \$100.

h) What are the breakeven prices; the price where Mr. Ryan makes zero profit?

The breakeven selling prices are \$50 and \$150.

Homework

1. Pythagorean Inc. develops a new calculator with voice recognition. The total profit y depends on the selling price x and is modeled by the equation:

$$y = -25(x - 100)(x - 200)$$

- How much profit will Pythagorean Inc. make if they sell the units for 175\$ each?
- What are the x -intercepts?
- Expand the equation to express it in standard form.
- What is the y -intercept?
- Determine the vertex of the above function.
- Use the above information to sketch a graph of Profit vs. Price.
- At what price will Pythagorean Inc. make the most profit and what will that profit be?
- What are the breakeven prices; the price where Pythagorean Inc. make zero profit?

2. A landscaping business is given 200m of fencing to create a rectangular enclosure around a garden to protect the vegetables from livestock. The area of the enclosure that can be made with 200 m fencing is given by the equation:

$$A = -(x - 0)(x - 100)$$

where

- A is the area of the enclosure in m^2
 - x is the width of the enclosure
 - $x - 100$ is the length of the enclosure
- Does the parabola defined by the equation above open up or down?
 - Is the vertex a minimum or a maximum?
 - What is the area of the rectangular enclosure if the width is 25 m?
 - What are the x -intercepts?
 - Determine the vertex of the above function.
 - Use the above information to sketch a graph of Area vs. Width.
 - What was the largest area of the rectangular enclosure that can be made with the 200m of fencing? What shape does this make?

Homework

$$\begin{aligned} \#1 \ a) \quad y &= -25(175-100)(175-200) \\ &= -25(75)(-25) \\ &= \$46875 \end{aligned}$$

b) x-ints: 100 & 200

$$\begin{aligned} c) \quad y &= -25(x-100)(x-200) \\ &= -25(x^2 - 200x - 100x + 20000) \\ &= -25(x^2 - 300x + 20000) \\ y &= -25x^2 + 7500x - 500000 \end{aligned}$$

d) y-int: -500000

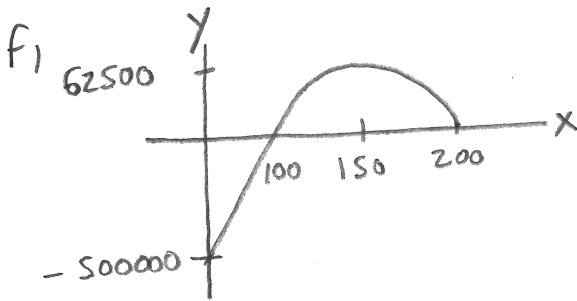
e) vertex

$$x = \frac{100+200}{2} \quad y = -25(150-100)(150-200)$$

$$x = 150 \quad y = -25(50)(-50)$$

$$y = 62500$$

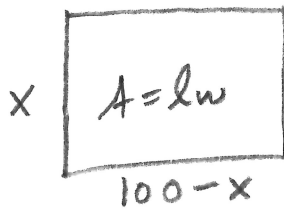
∴ The vertex is (150, 62500).



g) Pythagorean Inc. will make a maximum profit of \$62500 when the selling price is \$150.

h) The breakeven prices are \$100 and \$200

2.



$$A = x(100-x)$$

$$= (x-0)(-1)(x-100)$$

$$A = -(x-0)(x-100)$$

a) It opens down since there is a negative in front.

b) Since it opens down, the vertex will be a maximum.

c) set $x = 25$

$$A = -(25-0)(25-100)$$

$$= -(25)(-75)$$

$$= 1875 \text{ m}^2$$

d) x-ints: 0 & 100

e) vertex

$$x = \frac{0+100}{2}$$

$$x = 50$$

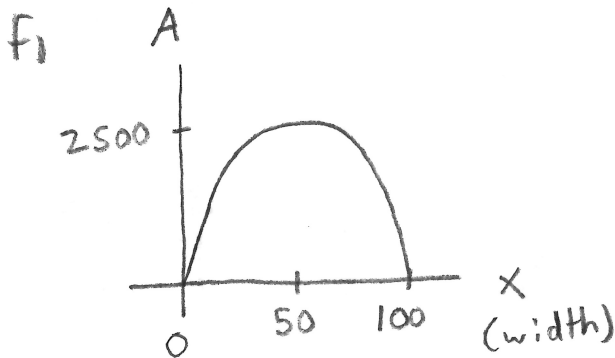
$$A = -(50-0)(50-100)$$

$$= -(50)(-50)$$

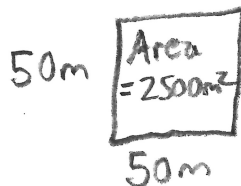
$$A = 2500$$

\therefore The vertex is $(50, 2500)$.

$x \quad A$



g) The largest rectangular enclosure that can be made with 200m of fencing has an area of 2500 m^2 ; see picture below



Shape is a square.