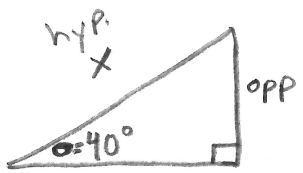
Trigonometry Review Worksheet

1a)



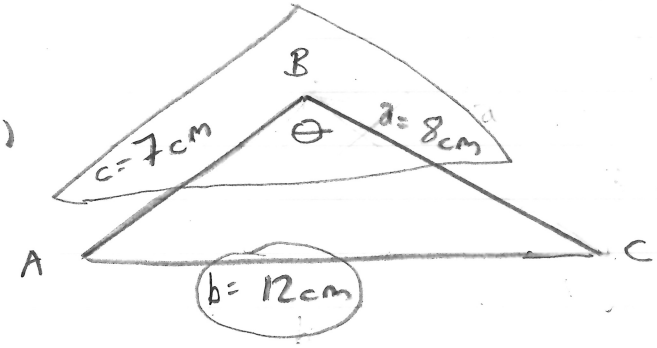
$\cos\theta = \frac{adj}{hyp}$

$\frac{\cos(40°)}{1} = \frac{5}{x}$

$\frac{x\cos(40°)}{\cos(40°)} = \frac{5}{\cos(40°)}$

$x \approx 6.5\text{ cm}$

b)



$b^2 = a^2 + c^2 - 2ac\cos B$

$12^2 = 8^2 + 7^2 - 2(8)(7)\cos\theta$

$144 = 64 + 49 - 112\cos\theta$

$112\cos\theta = 64 + 49 - 144$

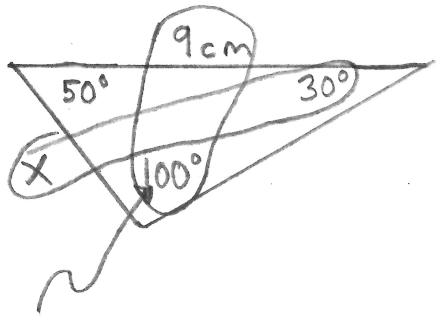
$\frac{112\cos\theta}{112} = \frac{-31}{112}$

$\cos\theta = \frac{-31}{112}$

$\theta = \cos^{-1}\left(\frac{-31}{112}\right)$

$\theta \approx 106.1°$

c)



$180° - 50° - 30°$

$= 100°$

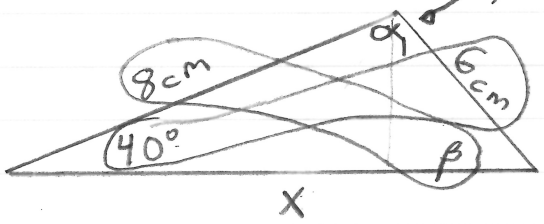
$\frac{\sin(100°)}{9} = \frac{\sin(30°)}{x}$

$\frac{x\sin(100°)}{\sin(100°)} = \frac{9\sin(30°)}{\sin(100°)}$

$x \approx 4.6\text{ cm}$

Ambiguous Case

2.



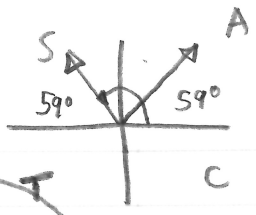
$180° - 40° - 59° = 81°$

$$\frac{\sin(40°)}{6} = \frac{\sin\beta}{8}$$

$$\frac{6\sin\beta}{6} = \frac{8\sin(40°)}{6}$$

$$\sin\beta = \frac{8\sin(40°)}{6}$$

$$\beta = \sin^{-1}\left(\frac{8\sin(40°)}{6}\right)$$



$\beta_1 = 59°$

$\alpha_1 = 81°$

$$\frac{\sin 40°}{6} = \frac{\sin(81°)}{x}$$

$$\frac{x\sin 40°}{\sin 40°} = \frac{6\sin(81°)}{\sin 40°}$$

$x_1 \cong 9.2\text{cm}$

$\beta_2 = 180° - 59° = 121°$

$\alpha_2 = 180° - 40° - 121° = 19°$

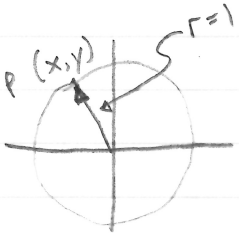
$$\frac{\sin 40°}{6} = \frac{\sin 19°}{x}$$

$$\frac{x\sin 40°}{\sin 40°} = \frac{6\sin 19°}{\sin 40°}$$

$x \cong 3\text{cm}$

* In the video solutions I accidentally used 59° instead of 81°.

3. a) $\theta = 135°$



$x = r\cos\theta$

$= 1\cos(135°)$

$x \cong -0.71$

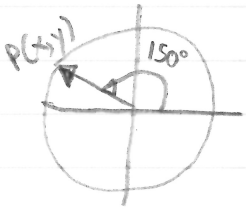
$y = r\sin\theta$

$= 1\sin(135°)$

$y \cong 0.71$

$P(x,y) \rightarrow (-0.71, 0.71)$

b) $\theta = 150°$



$x = r\cos\theta$

$= 1\cos(150°)$

$x \cong -0.87$

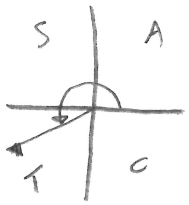
$y = r\sin\theta$

$= 1\sin(150°)$

$y = 0.5$

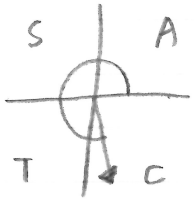
$P(x,y) \rightarrow (-0.87, 0.5)$

4. a) $\sin(200°)$



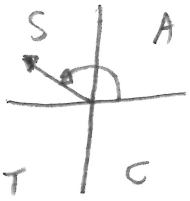
→ negative

b) $\cos(285°)$



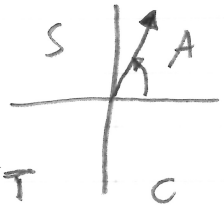
→ positive

c) $\tan(160°)$



→ negative

d) $\cos(60°)$

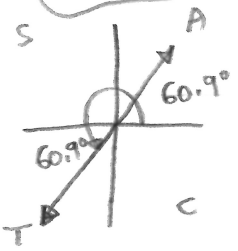


→ positive

5. a) $\tan\theta = 1.8$

$\theta = \tan^{-1}(1.8)$

$\theta_1 = 60.9°$



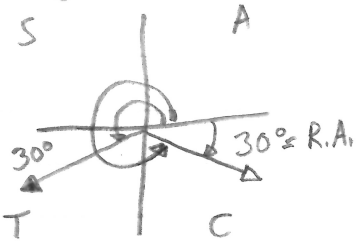
$\theta_2 = 180° + 60.9°$

$\theta_2 = 240.9°$

b) $\sin\theta = -0.5$

$\theta = \sin^{-1}(-0.5)$

$\theta = -30°$



$\theta_1 = 360° - 30°$

$\theta_1 = 330°$

$\theta_2 = 180° + 30°$

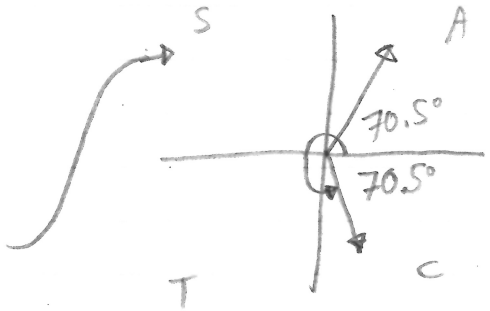
$\theta_2 = 210°$

c) $\sec\theta = 3$

$\cos\theta = \frac{1}{3}$

$\theta = \cos^{-1}\left(\frac{1}{3}\right)$

$\theta_1 \cong 70.5°$



$\theta_2 = 360° - 70.5°$

$\theta_2 \cong 289.5°$

d) $8\cos^2\theta - 2\cos\theta = 1$

$8\cos^2\theta - 2\cos\theta - 1 = 0$

Let $n = \cos\theta$

$8n^2 - 2n - 1 = 0$ $\quad P(-8)$, $S(-2)$ $\quad -4, 2$

$8n^2 - 4n + 2n - 1 = 0$

$4n(2n-1) + 1(2n-1) = 0$

$(4n+1)(2n-1) = 0$

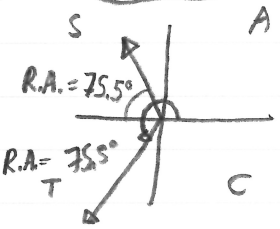
$n = -\frac{1}{4}$ or $n = \frac{1}{2}$

Case 1

$\cos\theta = -\frac{1}{4}$

$\theta = \cos^{-1}\left(-\frac{1}{4}\right)$

$\theta_1 \approx 104.5°$



$\theta_2 = 180° + 75.5°$

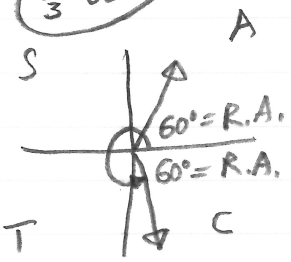
$\theta_2 \approx 255.5°$

Case 2

$\cos\theta = \frac{1}{2}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

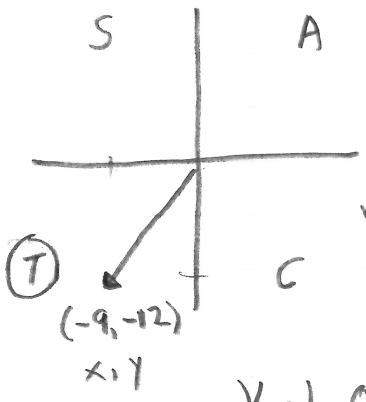
$\theta_3 = 60°$



$\theta_4 = 360° - 60°$

$\theta_4 = 300°$

6.



$r^2 = x^2 + y^2$

$r^2 = (-9)^2 + (-12)^2$

$r^2 = 81 + 144$

$\sqrt{r^2} = \sqrt{225}$

$r = 15$

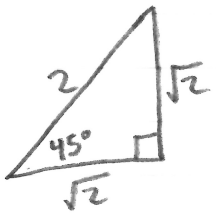
$\sin\theta = \frac{y}{r}$ $\quad \sin\theta = \frac{-12}{15}$ $\quad \sin\theta = -\frac{4}{5}$

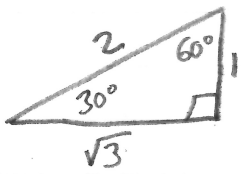
$\cos\theta = \frac{x}{r}$ $\quad \cos\theta = \frac{-9}{15}$ $\quad \cos\theta = -\frac{3}{5}$

$\tan\theta = \frac{y}{x}$ $\quad \tan\theta = \frac{-12}{-9}$ $\quad \tan\theta = \frac{4}{3}$

Yes! Only the tangent was positive.

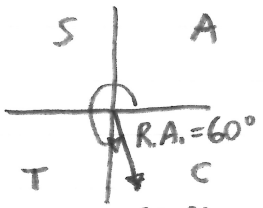
7.





a) $\tan(45°) = \frac{\sqrt{2}}{\sqrt{2}} = 1$

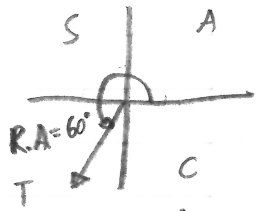
b) $\sin(300°)$



$= -\sin(60°)$

$= -\frac{\sqrt{3}}{2}$

c) $\cos(240°)$



$= -\cos(60°)$

$= -\frac{1}{2}$

8. $\tan^2\theta - \frac{1}{\cos^2\theta} = -1$

L.S. $\tan^2\theta - \frac{1}{\cos^2\theta}$ R.S. $= -1$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta - 1}{\cos^2\theta}$

$= \frac{-1 + \sin^2\theta}{\cos^2\theta}$

$= \frac{-(1 - \sin^2\theta)}{\cos^2\theta}$

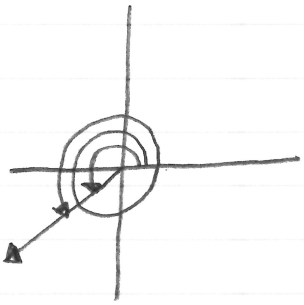
$= -\frac{\cos^2\theta}{\cos^2\theta}$

$= -1$

L.S. = R.S.

QED

9. a) $\theta_1 = 220°$



coterminal angles

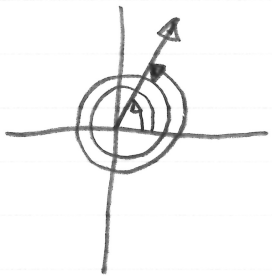
$\theta_2 = 360° + 220°$

$\theta_2 = 580°$

$\theta_3 = 220° - 360°$

$\theta_3 = -140°$

b) $\theta_1 = 60°$



$\theta_2 = 60° + 360° + 360°$

$\theta_2 = 780°$

$\theta_3 = 60° - 360°$

$\theta_3 = -300°$