Transformations of Sinusoidals: Part 2

The Effect of 'k' on Sinusoidals

For sinusoidal functions of the form:

$y = a\ sin[k(\theta - d)] + c$ or $y = a\ cos[k(\theta - d)] + c$

k→ Period compression/expansion (and a reflection about the y-axis)

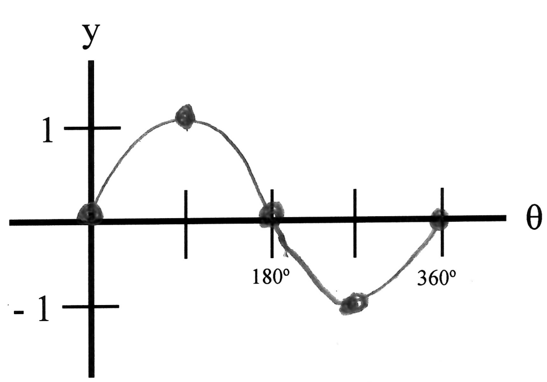
When |k| is larger than 1, the period of a sinusoidal is compressed.
When |k| is smaller than 1, the period of a sinusoidal is expanded.

When k is negative, the points (graph) is reflected about the y-axis.

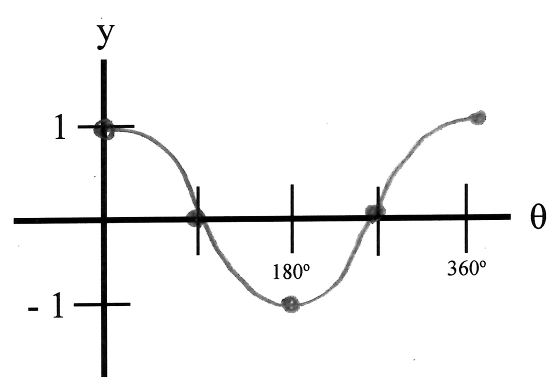
Recall

The 5 key points for a sinusoidal function are seen in the graphs below.

$y = \sin\theta$



$y = \cos\theta$



θ	$y = \sin\theta$
0°	0
90°	1
180°	0
270°	-1
360°	0

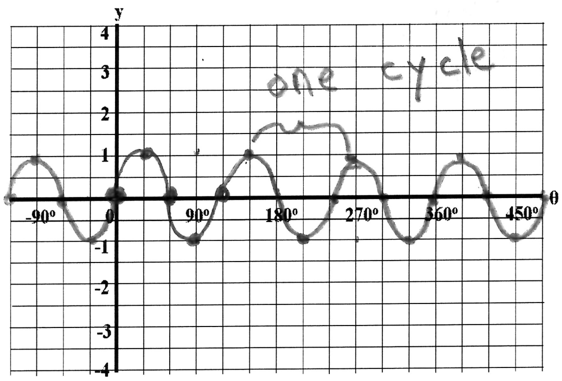
θ	$y = \cos\theta$
0°	1
90°	0
180°	-1
270°	0
360°	1

Example 1

Determine the value of k, the phase (d), then graph the function.
Use the graph to determine the period.

a) **y = sin(3θ)**

k = 3 d = ~~0~~

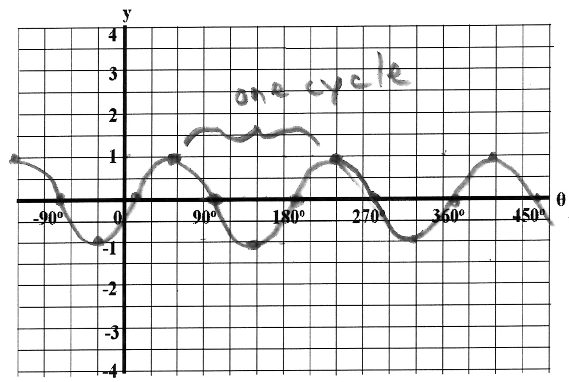


Period = 4 × 30°
= 120°

b) **y = cos(2θ – 120°)**

y = cos[2(θ – 60°)]

k = 2 d = 60°



Period = 6 × 30°
= 180°

The Relationship Between the Constant 'k' and the Period 'T'

In the first example above, the period was 120° and k = 3.
For the second example, the period was 180° and k = 2.

The constant k is related to the period, T, by the equations:

* can skip absolute value for applications

$$|k| = \frac{360^o}{T} \quad \text{or} \quad T = \frac{360^o}{|k|}$$

$k = \frac{360°}{T}$

Example 2

For each sinusoidal listed below, determine the period (T) and phase (d).

a) y = sin(4θ – 180°)

$\frac{-180°}{4} = -45°$

y = sin[4(θ – 45°)]

Period

$T = \frac{360°}{|k|}$

$= \frac{360°}{|4|}$

= 90°

d = 45°

Phase

b) y = cos(0.25θ + 15°)

y = cos[0.25(θ + 60°)]

$T = \frac{360°}{|0.25|}$

= 1440°

d = –60°

$\frac{15°}{0.25}$

$= \frac{15°}{1/4}$

$= 15° \div \frac{1}{4}$

$= \frac{15°}{1} \times \frac{4}{1}$

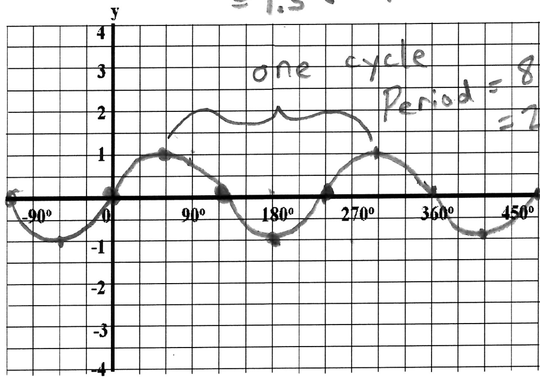
= 60°

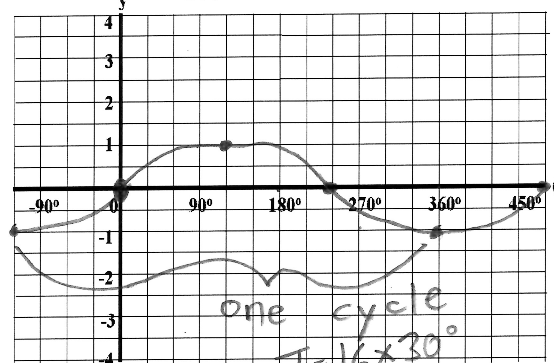
Example 3

Create a sinusoidal function of the form y = sinkθ then graph the function.

a) T = 240° $k = \frac{360°}{T} = \frac{360°}{240°} = 1.5 \leftarrow (3/2)$ $y = \sin(1.5\theta)$

b) T = 480° $k = \frac{360°}{T} = \frac{360°}{480°} = 0.75 \leftarrow (3/4)$ $y = \sin(0.75\theta)$





The Box Method for Graphing Sinusoidal Functions

The following procedure can be used to graph sinusoidal functions.

1. Extract the values of k, d, a, and c from the equation.
2. Draw a horizontal dotted line at $y = c$ to represent the line of equilibrium.
3. From the intersection of the y-axis and the line of equilibrium move left or right according to the phase shift, d; this is the left marker of the box.
4. Extend that marker up and done by the amplitude (amplitude = |a|). This represents the left side of the box.
5. Complete the box by extending to the right a width that corresponds to the period of the sinusoidal ($T = \frac{360°}{|k|}$).
6. Draw one cycle of the sinusoidal in the box taking the reflection values of k and a into consideration for reflections. Extend the pattern across the grid.

Example 4

use inverted cosine curve since 'a' is negative

Graph the sinusoidal functions using the box method; state the domain and range.

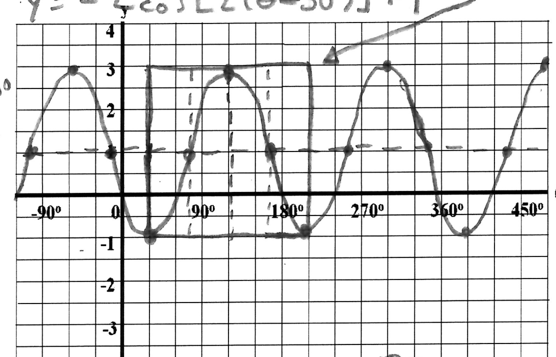
a) $y = -2\boxed{\cos}(2\theta - \cancel{30°}^{60°}) + 1$

$y = -2\cos[2(\theta - 30°)] + 1$

k = 2
d = 30°
a = −2
c = 1

amp = |a| = |−2| = 2

$T = \frac{360°}{|k|} = \frac{360°}{|2|} = 180°$



Domain: {θ ∈ ℝ}
Range: {y ∈ ℝ | −1 ≤ y ≤ 3}

b) $y = \boxed{\sin}(-\theta + 90°) - 2$

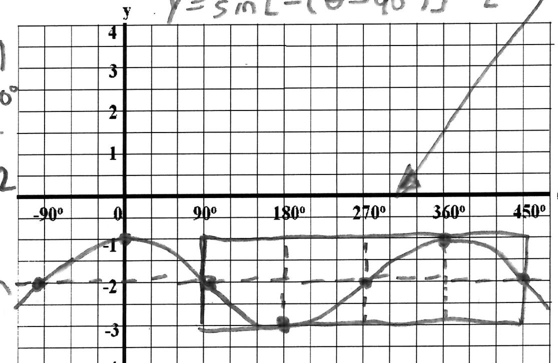
$y = \sin[-(\theta - 90°)] - 2$

k = −1
d = 90°
~~a = 1~~
c = −2

line of equilibrium

Amp = |a| = |1| = 1

$T = \frac{360°}{|k|} = \frac{360°}{|-1|} = 360°$



D: {θ ∈ ℝ}
R: {y ∈ ℝ | −3 ≤ y ≤ −1}

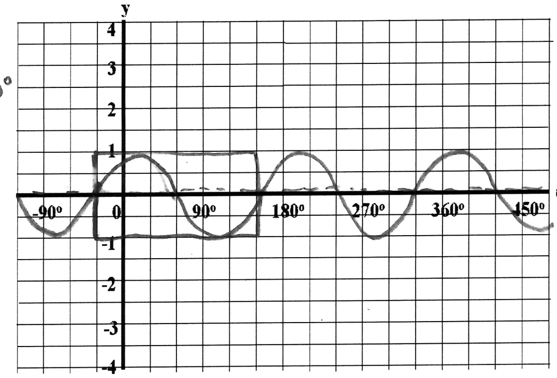
Homework: Complete graphs below + pg 379 # 1def, 2cdf, 3 Pg 383 #1acde, 9, 10

Use transformations to graph the following then state the domain and range.

a) $y = \sin(2\theta + 60^o)$

$y = \sin[2(\theta + 30°)] + 0$

$k = 2$
$d = -30°$
$\cancel{a = 1}$
$\cancel{c = 0}$
$T = \frac{360°}{|k|} = \frac{360°}{|2|} = 180°$
$Amp = |a| = 1$



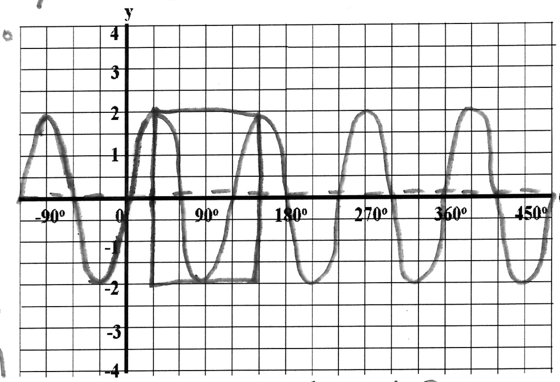
Domain: $\{\theta \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid -1 \le y \le 1\}$

b) $y = 2\cos(-3\theta + 90^o)$

$y = 2\cos[-3(\theta - 30°)] + 0$

$k = -3$
$d = 30°$
$a = 2$
$\cancel{c = 0}$
$T = \frac{360°}{|k|} = \frac{360°}{|-3|} = 120°$
$Amp = |a| = |2| = 2$

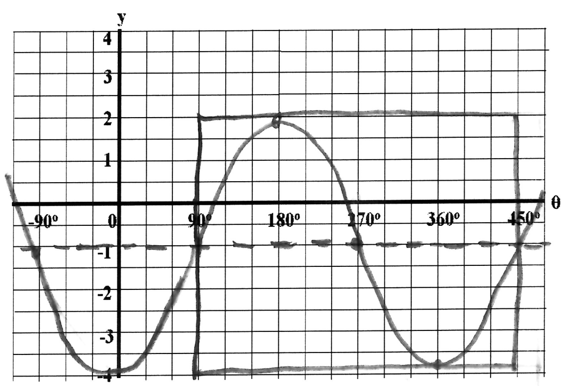


Domain: $\{\theta \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid -2 \le y \le 2\}$

c) $y = 3\sin(\theta - 90^o) - 1$

$\cancel{k = 1}$
$d = 90°$
$a = 3$
$c = -1$
$T = \frac{360°}{|k|} = \frac{360°}{|1|} = 360°$
$Amp = |a| = |3| = 3$



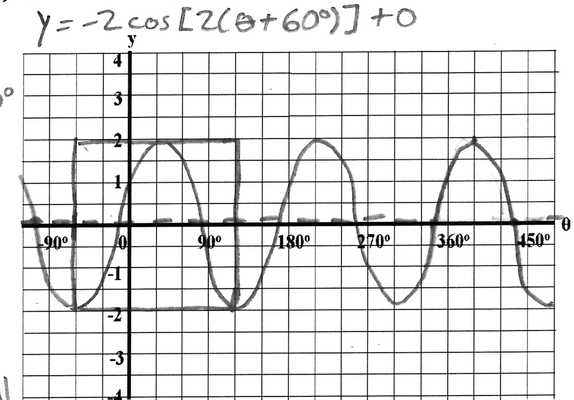
Domain : $\{\theta \in \mathbb{R}\}$

Range : $\{y \in \mathbb{R} \mid -4 \le y \le 2\}$

d) $y = -2\cos(2\theta + 120^o)$

$y = -2\cos[2(\theta + 60°)] + 0$

$k = 2$
$d = -60°$
$a = -2$
$\cancel{c = 0}$
$T = \frac{360°}{|k|} = \frac{360°}{|2|} = 180°$
$Amp = |a| = |-2| = 2$

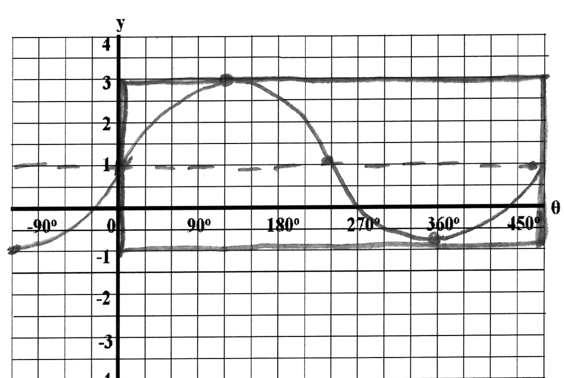


Domain: $\{\theta \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid -2 \le y \le 2\}$

e) $y = 2\sin(0.75\theta) + 1$

$k = 3/4$
$\cancel{d = 0}$
$a = 2$
$c = 1$
$T = \frac{360°}{|k|} = \frac{360°}{|3/4|} = 480°$
$Amp = |a| = |2| = 2$



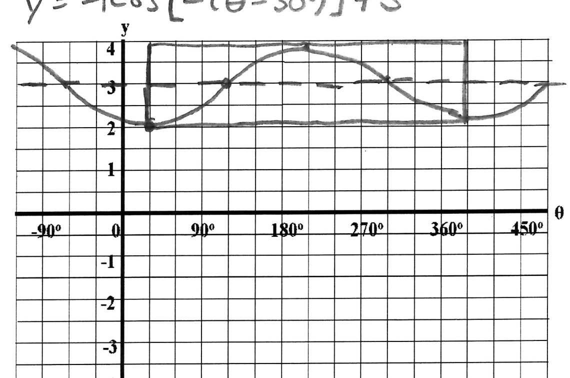
Domain: $\{\theta \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid -1 \le y \le 3\}$

f) $y = 3 - \cos(-\theta + 30^o)$

$y = -1\cos[-(\theta - 30°)] + 3$

$k = -1$
$d = 30°$
$a = -1$
$c = 3$
$T = \frac{360°}{|k|} = \frac{360°}{|-1|} = 360°$
$Amp = |a| = |-1| = -1$



Domain: $\{\theta \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid 2 \le y \le 4\}$