Transformations of Sinusoidals: Part 1

Recall:

vertical, follow intuition

Transformations of functions of the form $y = af[k(x - d)] + c$

horizontal, think opposite

k → horizontal compression/expansion and reflection
d → horizontal shift
a → vertical expansion/compression and reflection
c → vertical shift

Transformations of Sinusoidals

For sinusoidal functions of the form:

$y = a\ sin[k(\theta - d)] + c$ or $y = a\ cos[k(\theta - d)] + c$

k → period compression/expansion and reflection
d → phase shift

always positive

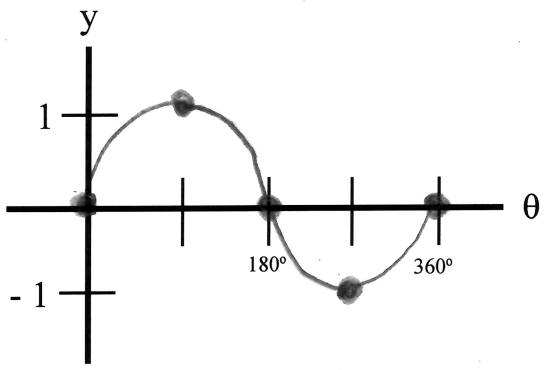
a → vertical expansion/compression and reflection (amplitude = $|a|$)
c → vertical displacement (location of the line of 'equilibrium' at y = c)

It is important to note that this extra terminology is reserved for use when discussing sinusoidal functions but the numerical constants (k, d, a, and c) continue to produce the same transformations as they have in previous units.

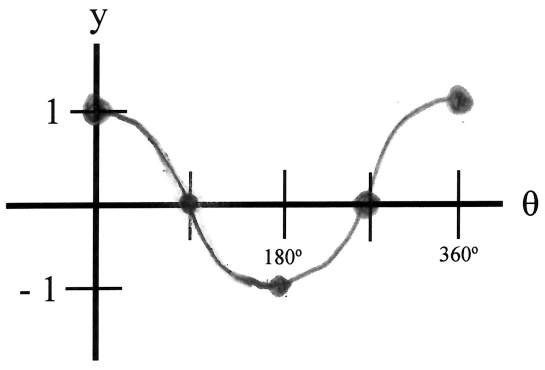
Activity

Create sketches of the parent functions $y = \sin\theta$ and $y = \cos\theta$.

$y = \sin\theta$



$y = \cos\theta$



The 5-point Method

To graph a sinusoidal function:

1. Apply transformations to the 5 key points from the relevant table below.

θ	y=sinθ
0°	0
90°	1
180°	0
270°	-1
360°	0

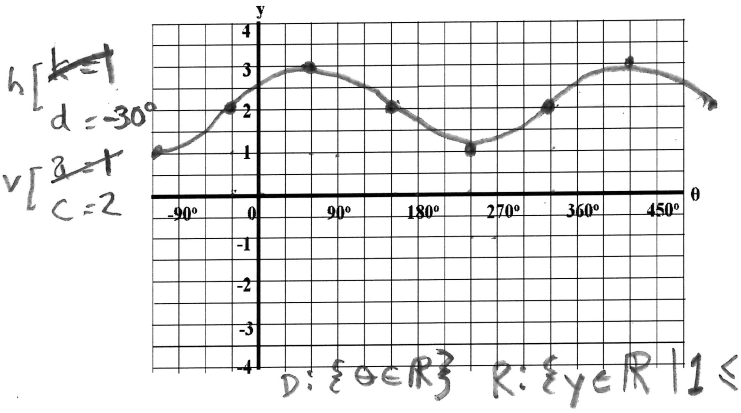
θ	y=cosθ
0°	1
90°	0
180°	-1
270°	0
360°	1

2. Extend the pattern beyond the 5 points on the graph then draw a smooth curve.

Examples

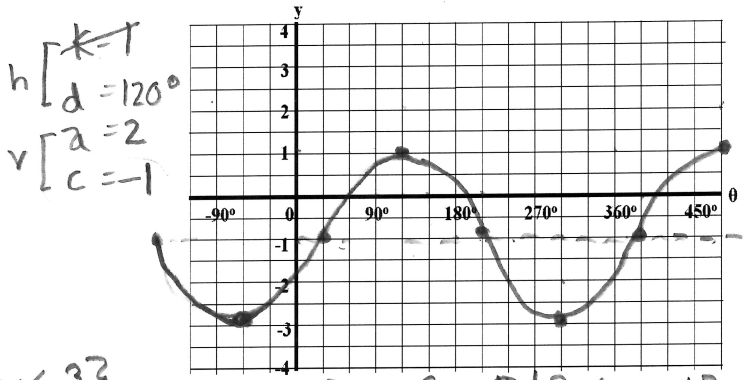
Graph the following then state the domain and range.

a) $y = \sin(\theta + 30°) + 2$



D: $\{\theta \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid 1 \leq y \leq 3\}$

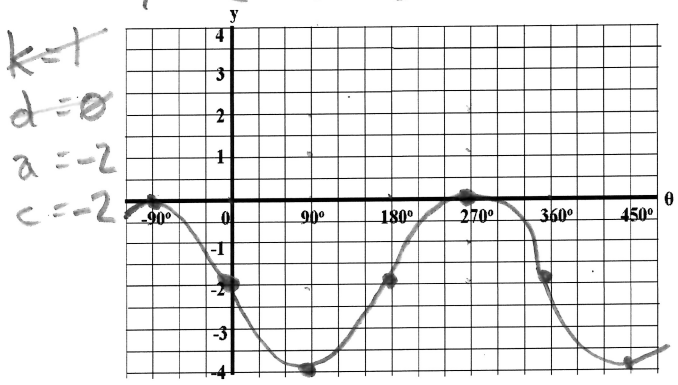
b) $y = 2\cos(\theta - 120°) - 1$



D: $\{\theta \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid -3 \leq y \leq 1\}$

c) $y = -2(\sin\theta + 1)$

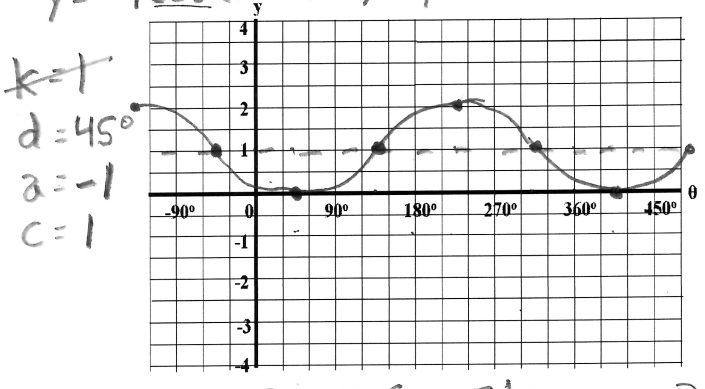
$y = -2\sin\theta - 2$



D: $\{\theta \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid -4 \leq y \leq 0\}$

d) $y = 1 - \cos(\theta - 45°)$

$y = -1\cos(\theta - 45°) + 1$

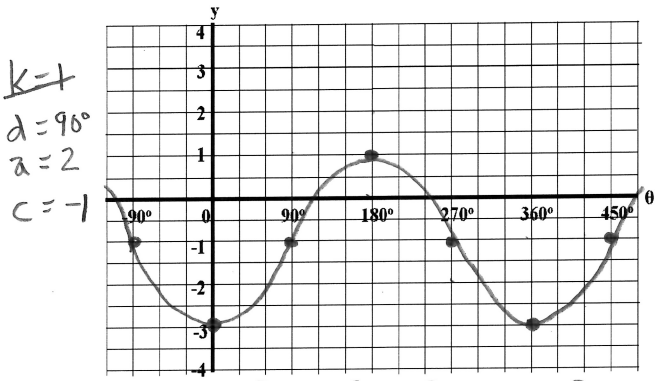


D: $\{\theta \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid 0 \leq y \leq 2\}$

Homework: Complete graphs below + pg 379 #1abc, 2abe, pg 383 # 1b, 4ad, 6ac

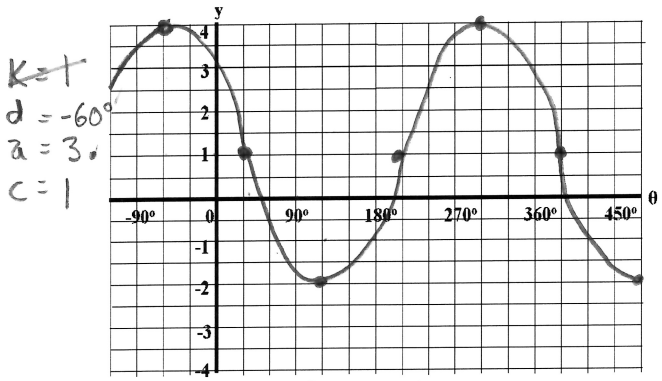
Use transformations to graph the following then state the domain and range.

a) $y = 2\sin(\theta - 90^\circ) - 1$



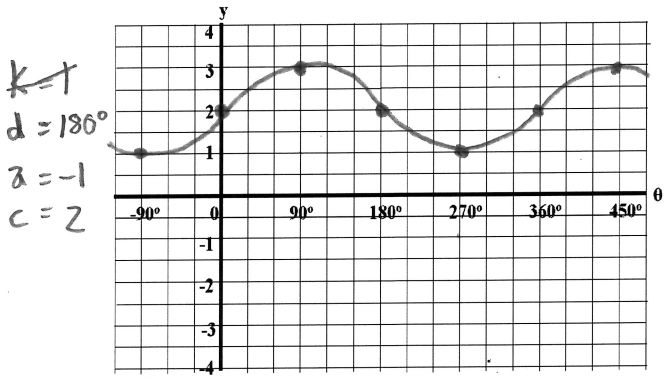
D: $\{\theta \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid -3 \le y \le 1\}$

b) $y = 3\cos(\theta + 60^\circ) + 1$



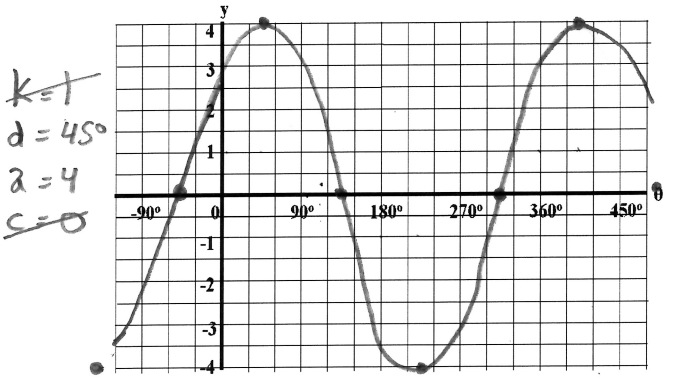
D: $\{\theta \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid -2 \le y \le 4\}$

c) $y = -\sin(\theta - 180^\circ) + 2$



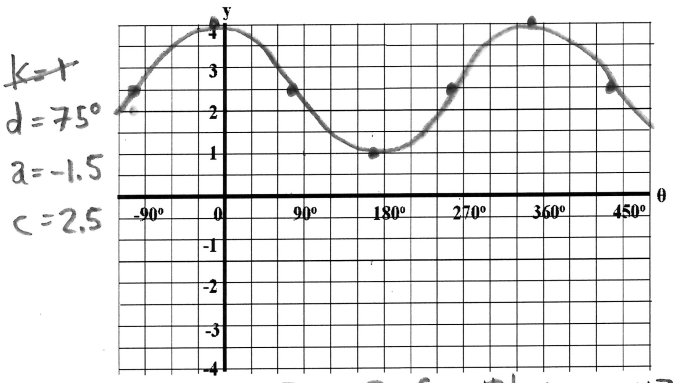
D: $\{\theta \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid 1 \le y \le 3\}$

d) $y = 4\cos(\theta - 45^\circ) + 0$



D: $\{\theta \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid -4 \le y \le 4\}$

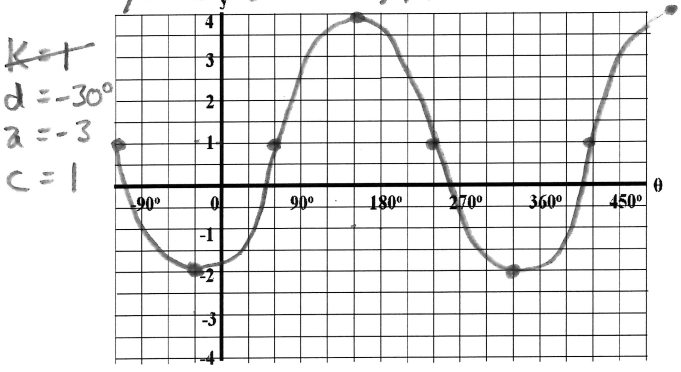
e) $y = -1.5\sin(\theta - 75^\circ) + 2.5$



D: $\{\theta \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid 1 \le y \le 4\}$

f) $y = 1 - 3\cos(\theta + 30^\circ)$

$y = -3\cos(\theta + 30^\circ) + 1$



D: $\{\theta \in \mathbb{R}\}$ R: $\{y \in \mathbb{R} \mid -2 \le y \le 4\}$