Transformations of Functions: Part 1

**Transformation –** a change in the shape/position of a graphed function resulting from a change to the equation of the function.

The graph of any function f(x) can undergo four transformations:

* Vertical Shift
* Vertical Expansion/Compression/Reflection
* Horizontal Shift
* Horizontal Expansion/Compression/Reflection

To better understand these transformations, let’s explore the effects of modifying the standard quadratic function f(x) = x2.

1. Vertical Shift

Graph these functions below.

f(x) = x2  g(x) = x2 - 4



**y**

**x**



**y**

**x**

The function g(x) resembles f(x) shifted down 4 units.

Since g(x) = f(x) – 4 or g(x) = f(x) + c where c = – 4, then, in general:

The curve y = f(x) + c is created by vertically shifting y = f(x) up ‘c’ units.

2. Vertical Expansion

Graph these functions below.

f(x) = x2  g(x) = 2x2



**y**

**x**



**y**

**x**

The function g(x) resembles f(x) but expanded vertically by a factor of 2. Since g(x) = 2f(x) or g(x) = af(x) where a = 2, then, in general:

The curve y = af(x) is created by vertically expanding y = f(x) by a factor of ‘a’.

3. Horizontal Shift

Graph these functions below.

f(x) = x2  g(x) = (x - 3)2



**y**

**x**



**y**

**x**

The function g(x) resembles f(x) shifted right 3 units.

Since g(x) = f(x-3) or g(x) = f(x - d) where p = 3, then, in general:

The curve y = f(x - d) is created by horizontally shifting y = f(x) right ‘d’ units.

4. Horizontal Expansion

Graph these functions below.

f(x) = x2  g(x) = (2x)2



**y**

**x**



**y**

**x**

The function g(x) resembles f(x) but compressed horizontally by a factor of ½ . Since g(x) = f(2x) or g(x) = f(kx) where k = 2, then, in general:

The curve y = f(kx) is created by horizontally expanding/compressing y = f(x) by a factor of ‘’.

* If , then the graph is compressed horizontally.
* If , then the graph is expanded horizontally.

# Summary

If we start with the graph of y = f(x) then:

* y = f(x) + c is vertically shifted up by ‘c’ units.
* y = af(x) is vertically expanded by a factor of ‘a’.
* y = f(x - d) is horizontally shifted right by ‘d’ units.
* y = f(kx) is horizontally expanded by a factor of ‘’.

## Activity

**Graph the functions on the left by using the table of values.**

**Graph the functions on the right using a transformation.**



**y**

**x**



**y**

**x**

**a)  **

|  |  |
| --- | --- |
| **x** | **y** |
| **-2** |  |
| **-1** |  |
| **0** |  |
| **1** |  |
| **2** |  |



**y**

**x**

**b)  **



**y**

**x**

|  |  |
| --- | --- |
| **x** | **y** |
| **0** |  |
| **1** |  |
| **4** |  |
| **9** |  |

**c)  **



**y**

**x**



**y**

**x**

|  |  |
| --- | --- |
| **x** | **y** |
| **-2** |  |
| **-1** |  |
| **-0** |  |
| **1** |  |
| **2** |  |