Transformations of Exponentials: Part 1

Recall:

Consider the parent function y = f(x).

If this function is modified using the constants k, d, a, and c as follows:

$$y = af[k(x - d)] + c$$

(inside: $k(x-d)$; outside: $af[\ldots]+c$)

Then the graph of this function will appear similar to the original function $y = f(x)$ with the following transformations:

- 'k' horizontally expands/compresses the graph by a factor of $\frac{1}{k}$.
- 'd' horizontally shifts the graph right 'd' units.
- 'a' vertically expands/compresses the graph by a factor of 'a'.
- 'c' vertically shifts the graph up 'c' units.

Example 1

Consider the parent function $y = \sqrt{x}$. Determine the values for the constants k, d, a, and c for each new relationship below. Describe each transformation.

a) $y = -3\sqrt{x+8} + 0$

b) $y = \sqrt{2x+10} + 8$

$y = 1\sqrt{2(x+5)} + 8$

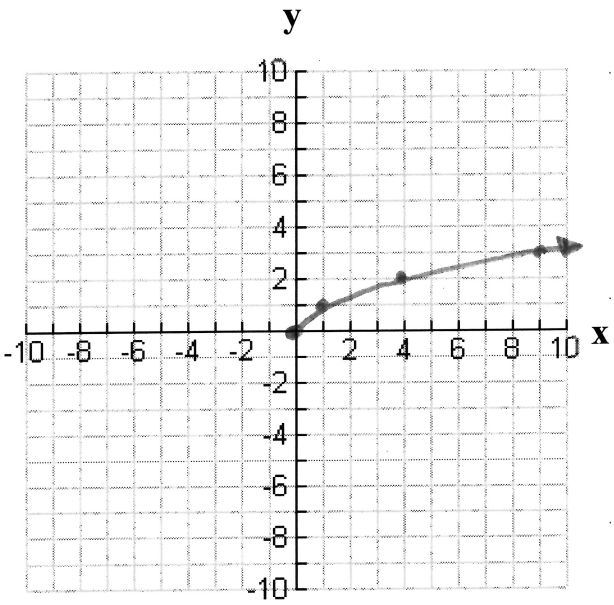
Constant	Transformation
k = ~~1~~	~~——————~~
d = −8	→ shift left 8 units
a = −3	→ vertical expansion by a factor of 3 → reflection about the x-axis
c = ~~0~~	~~——————~~

Constant	Transformation
k = 2	→ horizontal compression by a factor of 1/2
d = −5	→ shift left 5 units
a = ~~1~~	~~——————~~
c = 8	→ shift up 8 units

Example 2

a) Complete the table of values then graph the function $y = \sqrt{x}$.

x	y
0	0
1	1
4	2
9	3

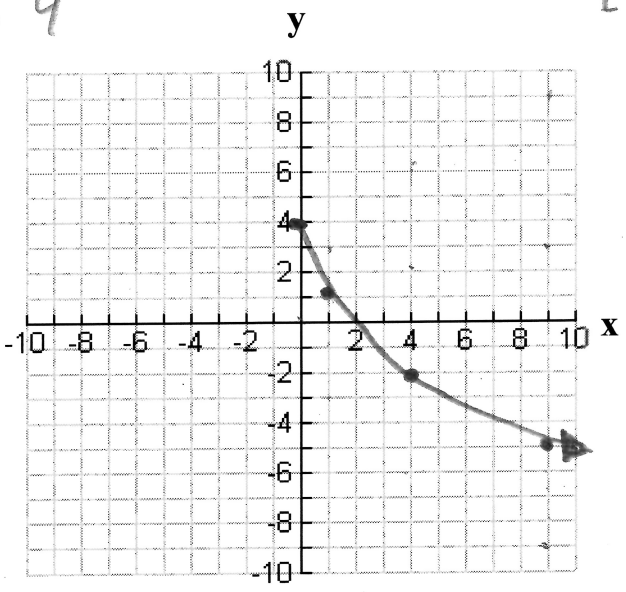


b) Use transformations to graph the following functions:

i) $y = -3\sqrt{x} + 4$

h [k = ~~1~~
  d = ~~0~~

v [a = -3
  c = 4

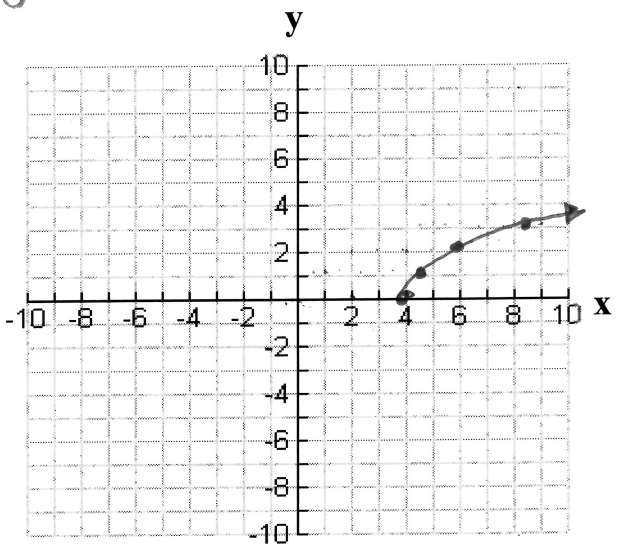


ii) $y = \sqrt{2x - 8}$

$y = \sqrt{2(x-4)}$

h [k = 2
  d = 4

v [~~a = 1~~
  ~~c = 0~~

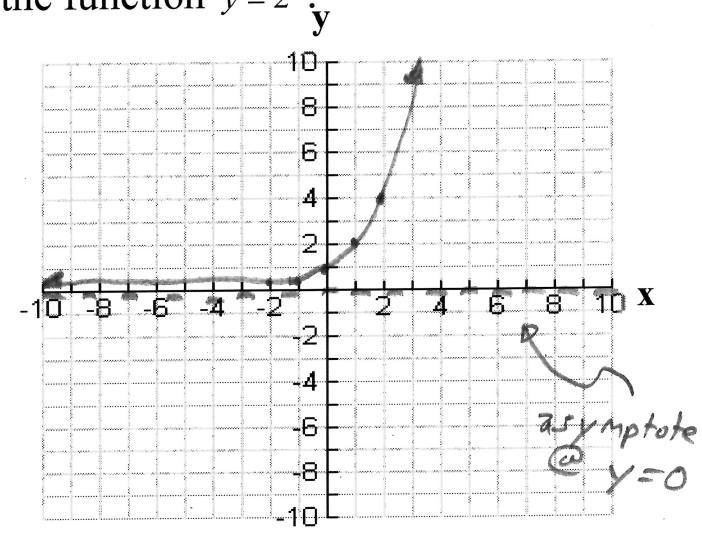


Example 3

a) Complete the table of values then graph the function $y = 2^x$.

x	$y = 2^x$
-2	1/4
-1	1/2
0	1
1	2
2	4

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$



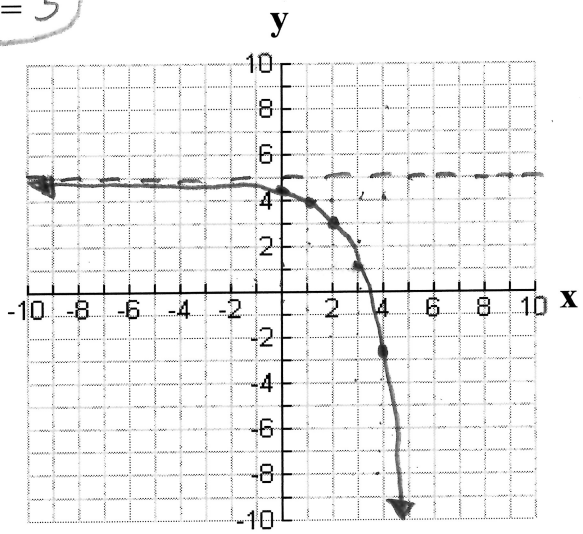
b) Use transformations to graph the following functions:

Note: For the parent function $y = 2^x$, the transformation constants are found in the following locations: $y = a(2)^{[k(x-d)]} + c$

i) $y = -2(2)^{x-2} + 5$

h [~~k = 1~~ ; d = 2

v [a = -2 ; c = 5] ← location of the H.A.



ii) $y = 2^{-\frac{1}{2}x-2}$

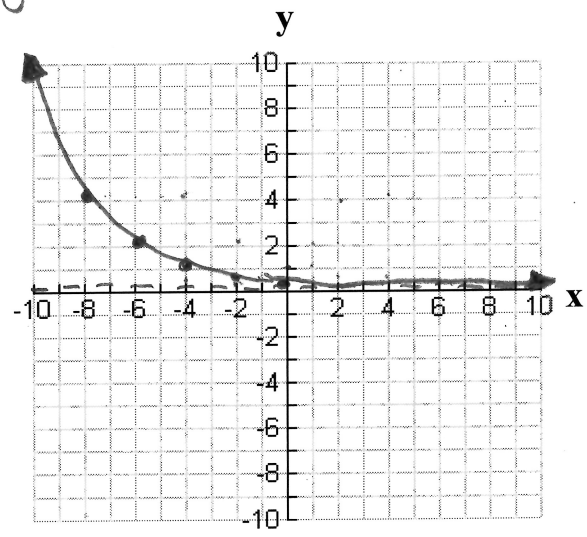
$y = 2^{-\frac{1}{2}(x+4)}$

$-2 \div -\frac{1}{2} = -2 \times -\frac{2}{1} = 4$

h [$k = -\frac{1}{2}$; d = -4

~~a = 1~~

~~c = 0~~



Four Possible Graphs