Transformations of Functions: Part 1

Transformation – a change in the shape/position of a graphed function resulting from a change to the equation of the function.

The graph of any function f(x) can undergo four transformations:

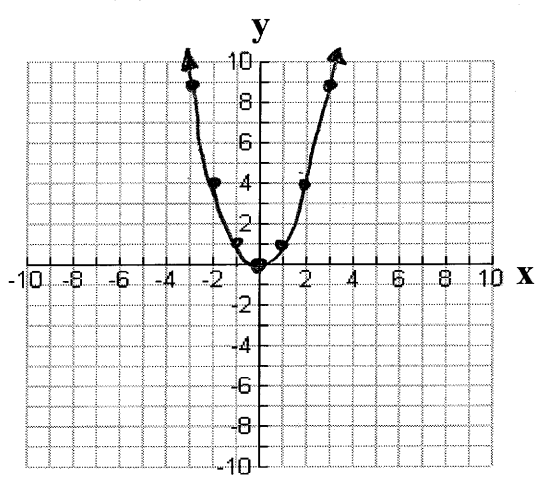
- Vertical Shift
- Vertical Expansion/Compression/Reflection
- Horizontal Shift
- Horizontal Expansion/Compression/Reflection

To better understand these transformations, let's explore the effects of modifying the standard quadratic function $f(x) = x^2$.

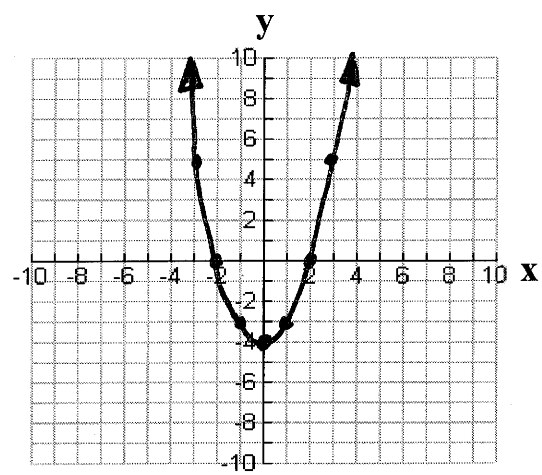
1. Vertical Shift

Graph these functions below.

$f(x) = x^2$



$g(x) = x^2 - 4$



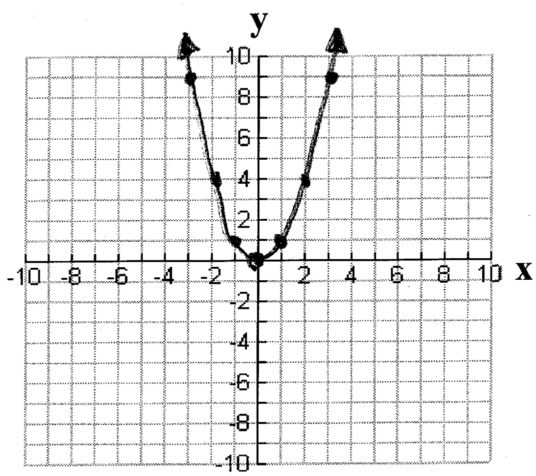
The function g(x) resembles f(x) shifted down 4 units.
Since g(x) = f(x) – 4 or g(x) = f(x) + c where c = – 4, then, in general:

The curve y = f(x) + c is created by vertically shifting y = f(x) up 'c' units.

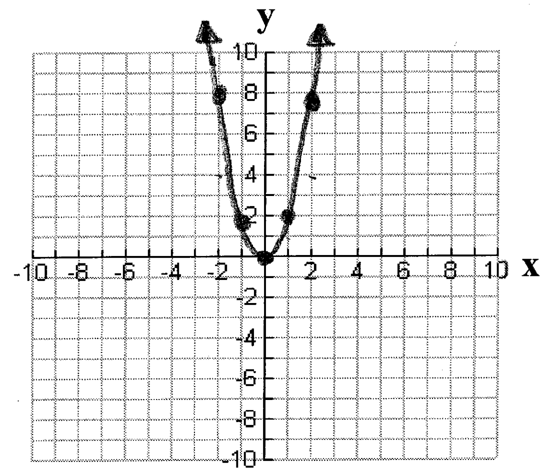
2. Vertical Expansion

Graph these functions below.

$f(x) = x^2$



$g(x) = 2x^2$



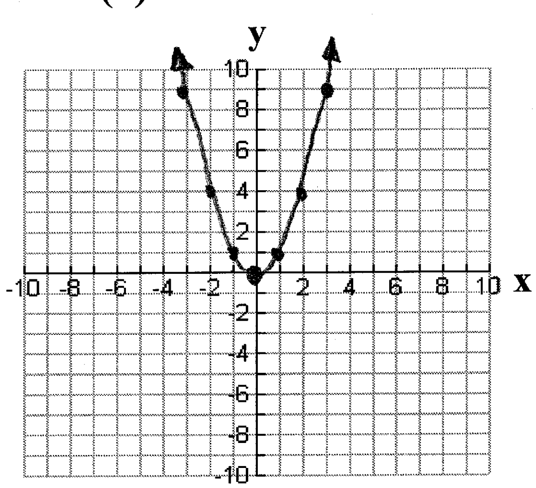
The function g(x) resembles f(x) but expanded vertically by a factor of 2. Since g(x) = 2f(x) or g(x) = af(x) where a = 2, then, in general:

The curve y = af(x) is created by vertically expanding y = f(x) by a factor of 'a'.

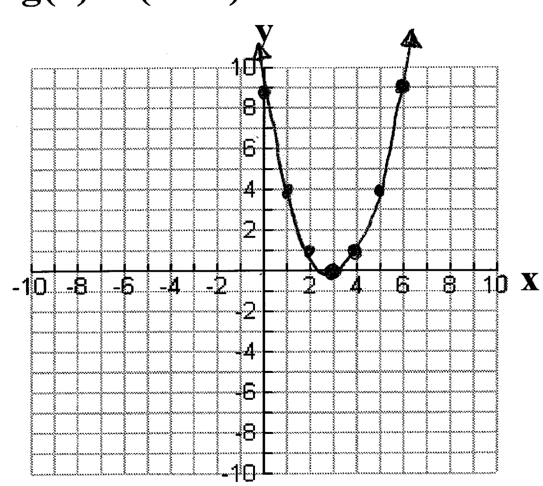
3. Horizontal Shift

Graph these functions below.

$\mathbf{f(x) = x^2}$



$\mathbf{g(x) = (x - 3)^2}$



The function g(x) resembles f(x) shifted right 3 units.
Since g(x) = f(x-3) or g(x) = f(x - d) where p = 3, then, in general:

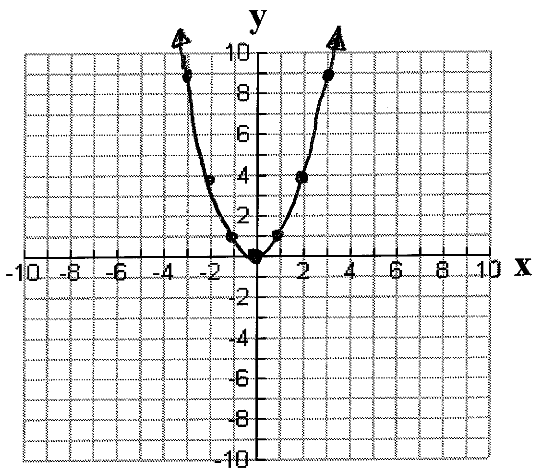
The curve y = f(x - d) is created by horizontally shifting y = f(x) right 'd' units.

$y = 2x^2$ ← 2 is outside

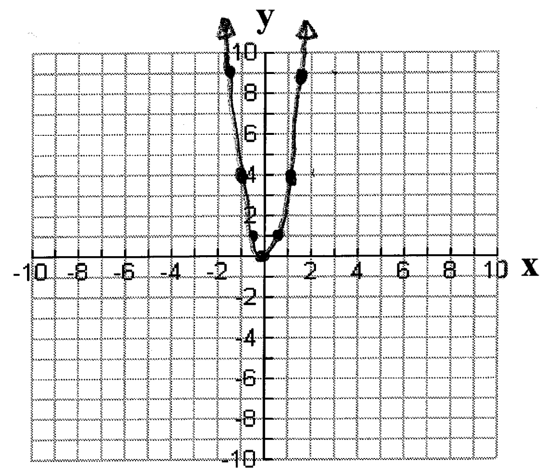
4. Horizontal Expansion

Graph these functions below.

$f(x) = x^2$



$g(x) = (2x)^2$ ← 2 is inside



The function g(x) resembles f(x) but compressed horizontally by a factor of ½ . Since g(x) = f(2x) or g(x) = f(kx) where k = 2, then, in general:

> The curve y = f(kx) is created by horizontally expanding/compressing y = f(x) by a factor of '$\frac{1}{k}$'.
>
> - If $|k| > 1$, then the graph is compressed horizontally.
> - If $|k| < 1$, then the graph is expanded horizontally.

Summary

> If we start with the graph of y = f(x) then:
>
> - y = f(x) + c is vertically shifted up by 'c' units.
> - y = af(x) is vertically expanded by a factor of 'a'.
> - y = f(x - d) is horizontally shifted right by 'd' units.
> - y = f(kx) is horizontally expanded by a factor of '$\frac{1}{k}$'.