Homework: pg 299 #5, 6iii, 8odd, 9odd, 12 and pg 304 – select additional practice

The "C.A.S.T." RULE

Recall: If P(x, y) is a point on the end of a terminal arm with angle in standard position θ, then the coordinates of this point can be expressed using the equations:

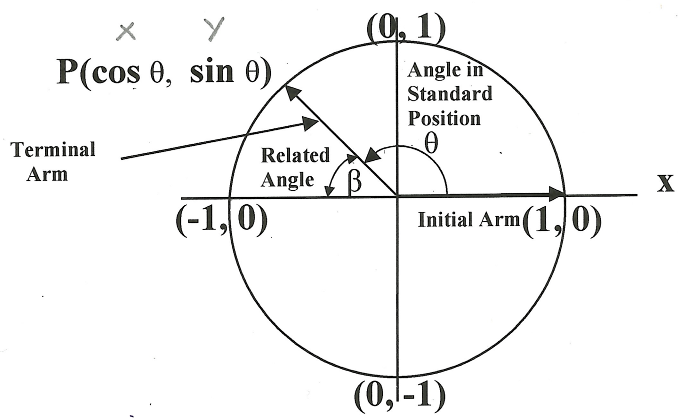
$x = r\cos\theta \qquad y = r\sin\theta$

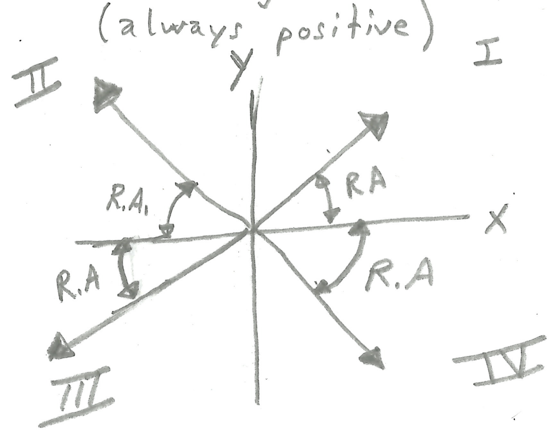
If P is a point on a unit circle then r = 1 and so the coordinates are given by

x = cosθ y = sinθ

This is shown in the diagram below.

Related Angles (always positive)





The angle between the terminal arm and the closest x-axis (positive or negative) is called the 'Related Acute Angle' or R. A. for short.

The C. A. S. T. Rule

Use your knowledge of the unit circle to determine the sign of sinθ, cosθ, and tanθ.

Quadrant II

x → −
y → +
r → +

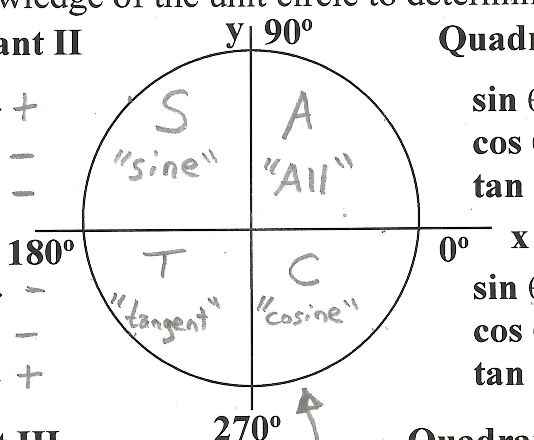
sin θ → +
cos θ → −
tan θ → −

Quadrant I

sin θ → + = +
cos θ → +
tan θ → +

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$



Quadrant III

x → −
y → −
r → +

sin θ → −
cos θ → −
tan θ → +

Quadrant IV

sin θ → −
cos θ → +
tan θ → −

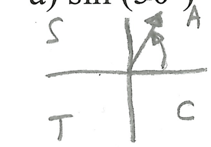
$\tan\theta = \frac{y}{x}$

What's positive in each quadrant?

Example 1

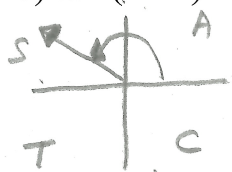
Use the CAST rule to determine the sign of each trigonometric function:

a) sin (50^0)



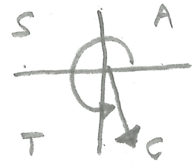
→ positive

b) tan (150^0)



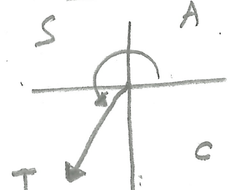
→ negative

c) cos (285^0)



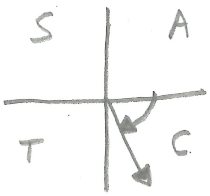
→ positive

d) sin (240^0)



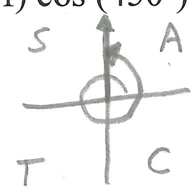
→ negative

e) tan (-50^0)



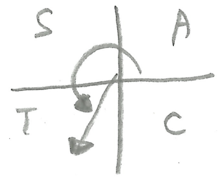
→ negative

f) cos (450^0)



$\frac{x}{r} = \frac{0}{\text{non-zero}} = 0$

g) sin (250^0)

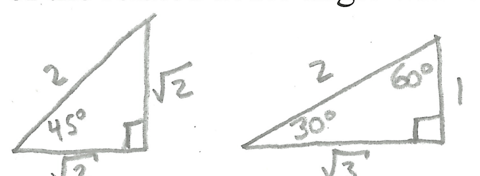


→ negative

h) tan (90°)

$\tan(90°) = \frac{y}{x} = \frac{\text{non-zero}}{0} = \text{undefined}$

The primary trigonometric ratio for any angle larger than 90° is equal to the trigonometric ratio of the related acute angle with the sign determined by the CAST rule.

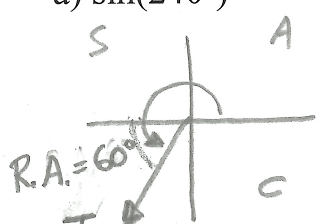


← special triangles

Example 2

Determine the trigonometric ratio in each case using the related acute angle and the CAST rule.

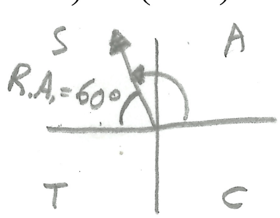
a) sin(240°)



$= -\sin(60°)$

$= -\frac{\sqrt{3}}{2}$

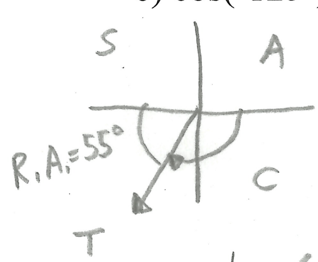
b) tan(120°)



$= -\tan(60°)$

$= -\sqrt{3}$

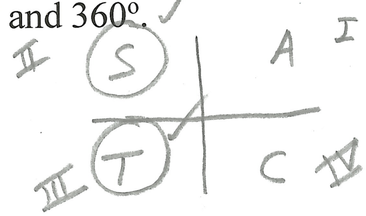
c) tan ~~cos~~(-125°)



$= \tan(55°)$

$\cong 1.428$

The CAST RULE can also help you solve a trigonometric equation for θ. There will (almost) always be two solutions for θ between 0° and 360°.



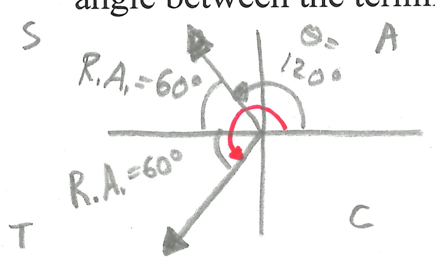
For example, when given an equation such as:

$$\cos\theta = -0.5$$

1. Identify which quadrants have cos θ equal to a negative value using the CAST rule; they are quadrant __II__ and quadrant __III__.
2. Use your calc, to get one of the answers.

$$\cos\theta = -0.5$$
$$\theta = \cos^{-1}(-0.5)$$
$$\theta_1 = 120°$$

3. Sketch ~~a unit circle with~~ a terminal arm using θ, the angle in standard position that you just calculated, and determine the related angle (the angle between the terminal arm and the x-axis). RA = __60°__.



4. On the diagram above, draw a related angle with a second terminal arm in the other location where cos θ is equal to a negative value.
5. The angle in standard position for this terminal arm is the other solution for θ.

$$\theta_2 = 180° + 60° = 240°$$

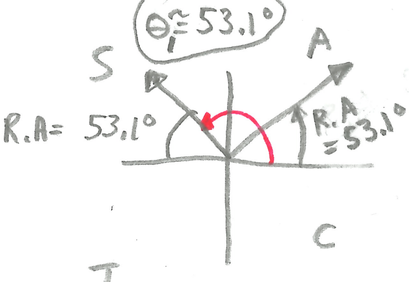
Example 3

Solve each equation below for the angle θ such that $0° \leq \theta \leq 360°$.

a) $\sin\theta = 0.8$

$\theta = \sin^{-1}(0.8)$

$\theta_1 \cong 53.1°$



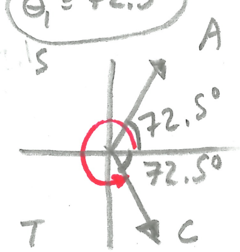
$\theta_2 = 180° - 53.1°$

$\cong 126.9°$

b) $\cos\theta = 0.3$

$\theta = \cos^{-1}(0.3)$

$\theta_1 \cong 72.5°$



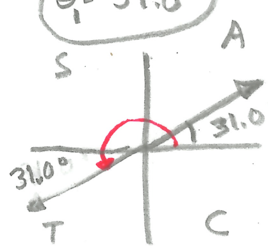
$\theta_2 = 360° - 72.5°$

$\cong 287.5°$

c) $\tan\theta = 0.6$

$\theta = \tan^{-1}(0.6)$

$\theta_1 = 31.0°$



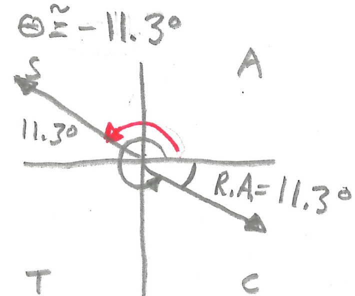
$\theta_2 = 180° + 31.0°$

$\cong 211.0°$

* d) $\tan\theta = -0.2$

$\theta = \tan^{-1}(-0.2)$

$\theta \cong -11.3°$



$\theta_1 = 360° - 11.3° \cong 348.7°$

$\theta_2 = 180° - 11.3° \cong 168.7°$