Homework: pg 287 # 4, 5ab, 6ab, 7abc, 8, 9, 10, 11

The Trigonometric Ratios of Special Angles

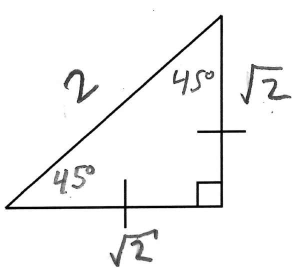
When a trigonometric function is computed in a calculator, the result is often an irrational number. For example, $\tan(60^o) = \sqrt{3}$. But $\sqrt{3}$ is an irrational number; it requires an infinite number of decimal places to represent this value exactly. A calculator will usually only list about 10 digits and so what is displayed is an ~~estimate~~. approximation.

Often, it is important to represent a value as an exact quantity instead of as an ~~estimate~~. approximation

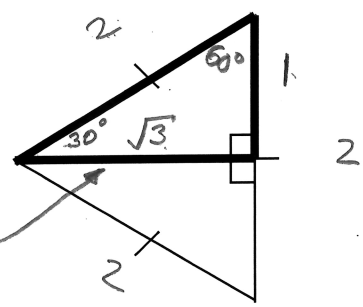
Exercise

Label all of the sides of the triangles below and use them to determine the exact values for sinθ, cosθ, and tanθ in the grid.

Isosceles Triangle



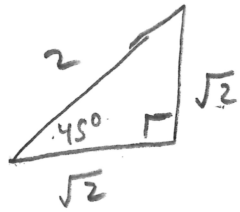
Equilateral Triangle

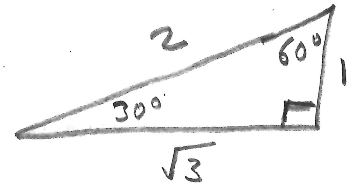


$a = \sqrt{c^2 - b^2}$
$= \sqrt{2^2 - 1^2}$
$= \sqrt{3}$

Trigonometric Function	θ = 30°	θ = 45°	θ = 60°
$\sin\theta = \frac{opp.}{hyp.}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta = \frac{adj.}{hyp.}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta = \frac{opp.}{adj.} = \frac{\sin\theta}{\cos\theta}$	$\frac{1}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\frac{\sqrt{2}}{\sqrt{2}} = 1$	$\frac{\sqrt{3}}{1} = \sqrt{3}$

$\sqrt{3}\sqrt{3} = \sqrt{9} = 3$





$1, 2, \sqrt{3}$

Example 1

Determine the exact value of the following expressions.

means $\cos(30°)\cos(30°)$

a) sin(60°) cos(45⁰) - sin(45°)

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{2\sqrt{2}}{4}$$

$$= \frac{\sqrt{6}-2\sqrt{2}}{4}$$

b) $\sin^2(30°)+\cos^2(30°)$

$$= \sin(30°)\sin(30°) + \cos(30°)\cos(30°)$$

$$= \frac{1}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

Example 2

Determine θ using the trigonometric ratio of special angles.

a) $\sqrt{2}\cos\theta = 1$

$$\cos\theta = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45°$$

b) $2\sin\theta = \sqrt{3}$

$$\frac{2\sin\theta}{2} = \frac{\sqrt{3}}{2}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 60°$$

Example 3

The sine double angle trigonometric identity states:

sin(2θ) = 2sinθcosθ

Proof that this is true when θ = 30°.

$$L.S. = \sin(2\theta)$$

$$= \sin[2(30°)]$$

$$= \sin[60°]$$

$$= \frac{\sqrt{3}}{2}$$

$$R.S. = 2\sin\theta\cos\theta$$

$$= 2\sin(30°)\cos(30°)$$

$$= \frac{\cancel{2}}{1}\cdot\frac{1}{\cancel{2}}\cdot\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2}$$

L.S. = R.S.

Q.E.D. (Quod Erat Demonstratum)