Exponential Relationships

Determine if each relationship is linear, quadratic, exponential or neither.

a)

x	y	1st diff.
3	22	-3
5	19	-3
7	16	-3
9	13	-3
11	10	

The 1st diff. in y are constant.
∴ Linear

b)

x	y	1st diff.	2nd diff.
-5	2	1	2
-3	3	3	2
-1	6	5	2
1	11	7	
3	18		

The 2nd diff. in y are constant.
∴ Quadratic

c)

x	y	1st diff.	ratios
8	3	2	$\frac{4}{2}=2$
5	5	4	2
2	9	8	2
-1	17	16	
-4	33		

The ratio of 1st diff. in y are constant.
∴ Exponential

Solving Simple Exponential Equations

Use a variety of techniques to solve the following equations.

a) $2^x = 32$

$2^x = 2^5$

$x = 5$

b) $8^{x+3} = 4^x$

$(2^3)^{(x+3)} = (2^2)^x$

$2^{3x+9} = 2^{2x}$

$3x+9 = 2x$

$3x-2x = -9$

$x = -9$

c) $243^{x-1} = 27^{x+3}$

$(3^5)^{(x-1)} = (3^3)^{(x+3)}$

$3^{5x-5} = 3^{3x+9}$

$5x-5 = 3x+9$

$5x-3x = 9+5$

$\frac{2x}{2} = \frac{14}{2}$ → $x = 7$

d) $2^{2x} - 7(2^x) - 8 = 0$

$(2^x)^2 - 7(2^x) - 8 = 0$

Let $n = 2^x$

$n^2 - 7n - 8 = 0$

$(n-8)(n+1) = 0$

$n = 8$ or $n = -1$

$2^x = 8$ $\quad$ $2^x = -1$

$2^x = 2^3$ $\quad$ No Sol^n

$x = 3$

e) $3^{2x} - 81 = 0$

$3^{2x} = 81$

$3^{2x} = 3^4$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

f) $3^x = 17$

$3^2 = 9$ and $3^3 = 27$

So $2 < x < 3$

Trial and error...

$3^{2.5} \approx 15.6$

$3^{2.8} \approx 21.7$

$x \approx 2.58$

Logarithms (Extension)

The log(x) determines the exponent that needs to be put on a base of '10' to make it equal to x.

Example 1

Evaluate each logarithm.

a) $\log(100)$ $10^? = 100$

$= 2$

b) $\log(1000)$ $10^? = 1000$

$= 3$

c) $\log(500)$ $10^? = 500$

≈ 2.70

d) $\log(0.01)$ $10^? = 0.01$, $10^? = \frac{1}{100}$

$= -2$

There are three rules that can be applied to logarithms:

Product Rule: $\log(AB) = \log(A) + \log(B)$

Quotient Rule: $\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$

* Power Rule: $\log(A)^n = n\log(A)$

Example 2

Solve the following exponential equations using logarithms.

a) $2^x = 32$

$\log 2^x = \log 32$

$\frac{x \log 2}{\log 2} = \frac{\log 32}{\log 2}$

$x = 5$

b) $3^x = 17$

$\log 3^x = \log 17$

$\frac{x \log 3}{\log 3} = \frac{\log 17}{\log 3}$

$x \approx 2.58$

c) $\frac{2(3)^x}{2} = \frac{100}{2}$

$3^x = 50$

$\log 3^x = \log 50$

$\frac{x \log 3}{\log 3} = \frac{\log 50}{\log 3}$

$x \approx 3.56$