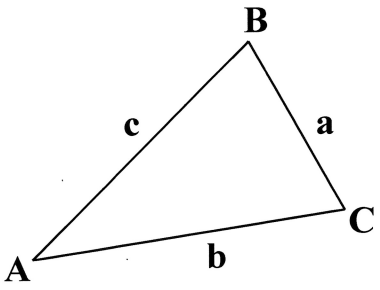
Homework: pg 318 # 1, 2, 4, 6, 8, pg 325 # 1, 2, 5, 6, 8

Sine Law and Cosine Law

Consider the triangle below.



Sine Law

→ used to determine a side or angle in a triangle when given a side across from an angle.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine Law

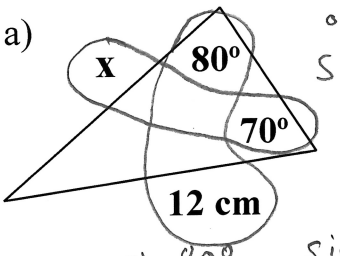
→ used to determine a side or angle in a triangle when working with two sides, its enclosed angle and the opposite side.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Example 1

Determine the unknown length of side x or angle θ as indicated in the following triangles.

a)



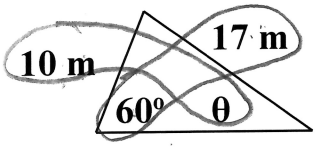
A-A-S or S-A-A

$$\frac{\sin 80^\circ}{12} = \frac{\sin 70^\circ}{x}$$

$$\frac{x \sin 80^\circ}{\sin 80^\circ} = \frac{12 \sin(70^\circ)}{\sin(80^\circ)}$$

$x \cong 11.45$ cm

b)



$$\frac{\sin(60^\circ)}{17} = \frac{\sin(\theta)}{10}$$

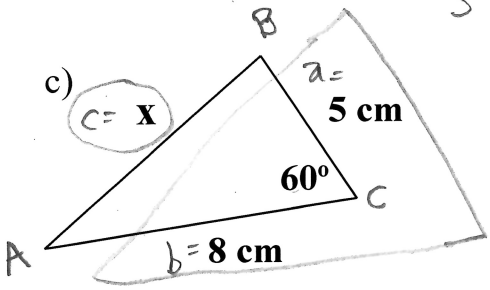
$$\frac{17 \sin\theta}{17} = \frac{10 \sin 60^\circ}{17}$$

$$\sin\theta = \frac{10 \sin 60^\circ}{17}$$

$$\theta = \sin^{-1}\left(\frac{10 \sin 60^\circ}{17}\right)$$

$\theta \cong 30.6^\circ$

c)



S-A-S

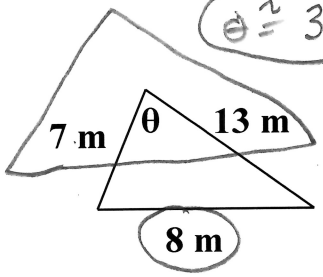
$c^2 = a^2 + b^2 - 2ab\cos C$

$x^2 \cong 5^2 + 8^2 - 2(5)(8)\cos 60^\circ$

$\sqrt{x^2} = \sqrt{49}$

$x = 7$ cm

d)



S-S-S

$8^2 = 7^2 + 13^2 - 2(7)(13)\cos\theta$

$64 = 49 + 169 - 182\cos\theta$

$182\cos\theta = 49 + 169 - 64$

$$\frac{182\cos\theta}{182} = \frac{154}{182}$$

$$\theta = \cos^{-1}\left(\frac{154}{182}\right)$$

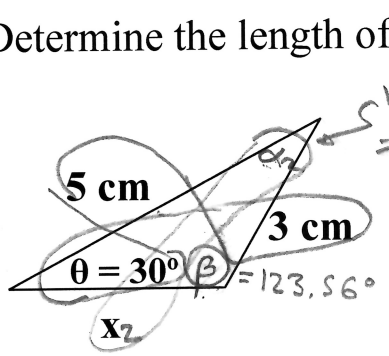
$\theta \cong 32.2^\circ$

The Ambiguous Case

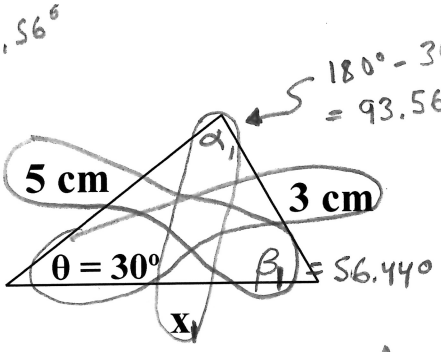
The ambiguous case arises in triangles when we are given the case Angle-Side$_1$-Side$_2$. If side$_2$ is smaller than side$_1$ then there may be two possible solutions for the triangle. There are two ways to address this scenario

Example 2

Determine the length of side x.



or



Solution 1 (Using Sine Law)

$$\frac{\sin 30°}{3} = \frac{\sin\beta}{5}$$

$$\frac{3\sin\beta}{3} = \frac{5\sin(30°)}{3}$$

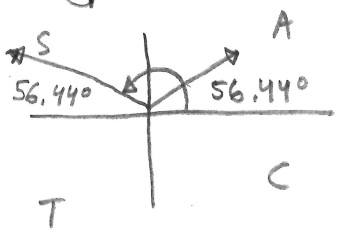
$$\beta = \sin^{-1}\left(\frac{5\sin(30°)}{3}\right)$$

$$\beta_1 = 56.44°$$

$$\frac{\sin(30°)}{3} = \frac{\sin(93.56°)}{x}$$

$$\frac{x\sin(30°)}{\sin(30°)} = \frac{3\sin(93.56°)}{\sin(30°)}$$

$$x_1 \cong 5.99\text{cm}$$



$$\beta_2 = 180° - 56.44°$$

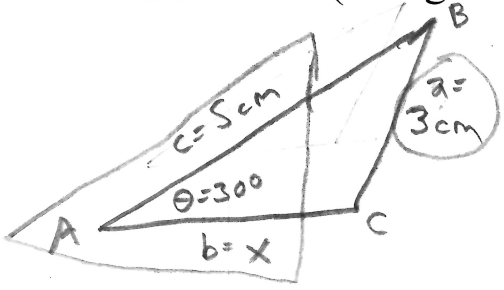
$$\beta_2 \cong 123.56°$$

$$\frac{\sin(30°)}{3} = \frac{\sin(26.44°)}{x}$$

$$\frac{x\sin(30°)}{\sin(30°)} = \frac{3\sin(26.44°)}{\sin(30°)}$$

$$x_2 \cong 2.67\text{ cm}$$

Solution 2 (Using Cosine Law)



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$3^2 = x^2 + 5^2 - 2(x)(5)\cos 30°$$

$$9 = x^2 + 25 - 8.66x$$

$$0 = x^2 - 8.66x + 16$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{8.66 \pm \sqrt{(-8.66)^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{8.66 \pm 3.32}{2}$$

$x_1 \cong 5.99$ cm

or

$x_2 \cong 2.67$cm