

Evaluating and Simplifying Rational Expressions

Rational Expression – any quantity that can be expressed in the form $\frac{p}{q}$
 where p and q are polynomials and/or integers and
 $q \neq 0$.

Ex; $\frac{4x+7}{3x+2}$, $\frac{x^3-125}{1}$, $\frac{2}{7}$, 5.32 , etc, ...

$= \frac{5.32}{1} = \frac{532}{100}$

The constant π would be a non-example since $\pi = 3.141592\dots$ cannot be written in the form $\frac{p}{q}$ with 'p' as an integer.

↑ irrational
 Ex: $\sqrt{2}, \sqrt{3}, \sqrt{5}$

Example 1Evaluate $f(3)$.

a) $f(x) = \frac{x^2+1}{x+2}$

$$\begin{aligned} f(3) &= \frac{(3)^2+1}{(3)+2} \\ &= \frac{9+1}{5} \\ &= \frac{10}{5} \\ &= 2 \end{aligned}$$

b) $f(x) = \frac{-x^2+4x+2}{x^2+1}$

$$\begin{aligned} f(3) &= \frac{-(3)^2+4(3)+2}{(3)^2+1} \\ &= \frac{-9+12+2}{9+1} \\ &= \frac{5}{10} \\ &= \frac{1}{2} \end{aligned}$$

* c) $f(x) = \frac{x+5}{x-3}$

$$\begin{aligned} f(3) &= \frac{(3)+5}{(3)-3} \\ &= \frac{8}{0} \quad \leftarrow \text{non-zero} \\ &\text{Undefined} \end{aligned}$$

Notice in example c) that $f(3)$ cannot be evaluated since the denominator is equal to zero when $x = 3$. For $f(x) = \frac{x+5}{x-3}$, x cannot be assigned the value 3. This is called a restriction and so we state that $x \neq 3$.

Example 2

State the restrictions for the following rational expressions.

a) $\frac{x-3}{x-2}$
 $x \neq 2$

b) $\frac{2x^2-5x+2}{x}$
 $x \neq 0$

c) $\frac{x^2+3}{(2x-5)(x+3)}$
 $x \neq -3$

d) $\frac{3x^2}{x-y}$
 $x-y \neq 0$
 $x \neq y$
 relation restriction

e) $\frac{5x^2-y+2}{8}$
 No restriction

$2x-5 \neq 0$
 $\frac{2x}{2} \neq \frac{5}{2}$
 $x \neq \frac{5}{2}$

Example 3

Simplify and state the restrictions.

a) $\frac{5x+25}{-3x-15}$
 $= \frac{5(\cancel{x+5})}{-3(\cancel{x+5})}$
 $= -\frac{5}{3}, x \neq -5$

b) $\frac{x^2-9}{2x-6}$
 $= \frac{(\cancel{x-3})(x+3)}{2(\cancel{x-3})}$
 $= \frac{x+3}{2}, x \neq 3$

c) $\frac{2x+5}{x^2+5x-14}$
 $= \frac{2x+5}{(x+7)(\cancel{x-2})}$
 $x \neq -7, 2$

d) $\frac{2m^2-18}{m^2-6m+9}$
 $= \frac{2(m^2-9)}{(m-3)(m-3)}$
 $= \frac{2(\cancel{m-3})(m+3)}{(\cancel{m-3})(m-3)}$
 $= \frac{2(m+3)}{m-3}$
 $m \neq 3$

e) $\frac{2x^2-7x-15}{8x+12}$
 $= \frac{(\cancel{2x+3})(x-5)}{4(\cancel{2x+3})}$
 $= \frac{x-5}{4}, x \neq -\frac{3}{2}$

f) $\frac{-8x^3y^4}{24x^4y^3}$
 $= -\frac{y}{3x}, y \neq 0$
 $x \neq 0$

$P(-30) \begin{cases} -10, 3 \\ 5(-7) \end{cases}$

$2x^2-7x-15 = 2x^2-10x+3x-15 = 2x(x-5)+3(x-5) = (2x+3)(x-5)$