

Simplifying Exponential Expressions

Example 1

A microorganism's ability to survive is proportional to its surface area to volume ratio. The shape of many microorganisms can be approximated by a sphere. Determine the ratio of the surface area of a sphere to its volume. Which microorganisms have a greater chance of survival? Large or small?

$$\begin{aligned} \text{Microorganism's Survival Ability} &\sim \frac{\text{Surface area of sphere}}{\text{Volume of sphere}} \\ &\sim \frac{4\pi r^2}{\frac{4\pi r^3}{3}} \\ &\sim \frac{4\pi r^2}{1} \div \frac{4\pi r^3}{3} \end{aligned} \quad \left. \begin{aligned} &\sim \frac{4\pi r^2}{1} \times \frac{3}{4\pi r^3} \\ &\sim \frac{3r^2}{r^3} \\ &\sim \frac{3}{r} \end{aligned} \right\}$$

∴ Smaller microorganisms have a greater ability to survive.

Example 2

Evaluate the expression $\frac{x^{2y-3}x^{y+1}}{x^{y-4}}$ when $x = -2$ and $y = 1$.

Method 1

$$\begin{aligned} &\frac{x^{2y-3} x^{y+1}}{x^{y-4}} \\ &= \frac{(-2)^{2(1)-3} (-2)^{(1)+1}}{(-2)^{(1)-4}} \\ &= \frac{(-2)^{-1} (-2)^2}{(-2)^{-3}} \\ &= \frac{(-2)^1}{(-2)^{-3}} \\ &= (-2)^{1-(-3)} \\ &= (-2)^4 \\ &= 16 \end{aligned}$$

Method 2

$$\begin{aligned} &\frac{x^{2y-3} x^{y+1}}{x^{y-4}} \\ &= \frac{x^{3y-2}}{x^{y-4}} \\ &= x^{(3y-2)-(y-4)} \\ &= x^{3y-2-y+4} \\ &= x^{2y+2} \\ &= (-2)^{2(1)+2} \\ &= (-2)^4 \\ &= 16 \end{aligned}$$

Example 3

Simplify the expression $\frac{(3x^{-1}y^2)^3}{(2x^2y^{-1})^2}$

$$\begin{aligned} &= \frac{(3)^3 (x^{-1})^3 (y^2)^3}{(2)^2 (x^2)^2 (y^{-1})^2} \\ &= \frac{27/x^{-3} y^6}{4x^4 y^{-2}} \\ &= \frac{27y^8}{4x^7} \end{aligned}$$

Example 4

Simplify the expression $\frac{(8x^6y^{-3})^{\frac{1}{3}}}{(16x^4y^{-2})^{\frac{1}{2}}}$

$$\begin{aligned} &= \frac{(8)^{\frac{1}{3}} (x^6)^{\frac{1}{3}} (y^{-3})^{\frac{1}{3}}}{(16)^{\frac{1}{2}} (x^4)^{\frac{1}{2}} (y^{-2})^{\frac{1}{2}}} \\ &= \frac{\sqrt[3]{8} x^2 y^{-1}}{\sqrt{16} x^2 y^{-1}} \end{aligned}$$

$= \frac{2}{4}$
 $= \frac{1}{2}, x \neq 0, y \neq 0$

Example 5

Simplify the expression $\left(\frac{\sqrt[3]{x^2}}{\sqrt{x^5}}\right)^4$

$$\begin{aligned} &= \left(\frac{x^{\frac{2}{3}}}{x^{\frac{5}{2}}}\right)^4 \\ &= \frac{(x^{\frac{2}{3}})^4}{(x^{\frac{5}{2}})^4} \end{aligned}$$

$= \frac{x^{\frac{8}{3}}}{x^{\frac{20}{2}}}$
 $= x^{\frac{8}{3} - \frac{30}{3}}$
 $= x^{-\frac{22}{3}}$
 $= \frac{1}{x^{\frac{22}{3}}}$
 $= \frac{1}{(\sqrt[3]{x})^{22}}$