Soln

Sequences and Series – Unit Practice

1. Expand the following binomial; you may use Pascal's Triangle if needed.

$(2x-3y^2)^4 = 1(2x)^4(-3y^2)^0 + 4(2x)^3(-3y^2)^1 + 6(2x)^2(-3y^2)^2 + 4(2x)^1(-3y^2)^3 + 1(2x)^0(-3y^2)^4$

$= 1(16x^4)(1) + 4(8x^3)(-3y^2) + 6(4x^2)(9y^4) + 4(2x)(-27y^6) + 1(1)(81y^8)$

$= 16x^4 - 96x^3y^2 + 216x^2y^4 - 216xy^6 + 81y^8$

2. Consider the sequence: 2, 5, 8, …, 365. arithmetic $a=2$ $d=3$

a. Determine the 89th term.

$t_n = a+(n-1)d$

$t_n = 2+(n-1)3$

$t_n = 2+3n-3$

$t_n = -1+3n$

set $n=89$

$t_{89} = -1+3(89)$

$t_{89} = 266$

b. Determine the sum of the terms in this sequence.

$t_n = -1+3n$

Set $t_n = 365$

$365 = -1+3n$

$\frac{366}{3} = \frac{3n}{3}$

$n = 122$

$S_n = \frac{n[2a+(n-1)d]}{2}$

$S_{122} = \frac{122[2(2)+(121)(3)]}{2}$

$S_{122} = \frac{122[367]}{2}$

$S_{122} = 22387$

3. Consider the sequence: 3, -6, 12, -24,.... geometric $a=3$ $r=-2$

a. Determine the 21st term.

$t_n = ar^{n-1}$

$t_n = 3(-2)^{n-1}$

set $n=21$

$t_{21} = 3(-2)^{20}$

$t_{21} = 3145728$

b. Determine the sum of the terms in the sequence up to the 24th term.

$S_n = \frac{a(r^n-1)}{r-1}$

$S_{24} = \frac{3[(-2)^{24}-1]}{(-2)-1}$

$S_{24} = -16777215$

4. Create a <u>recursive formula</u> and <u>general term</u> formula for the following sequences:

a. 54, 18, 6,… ← geometric $a=54$ $r=\frac{1}{3}$

Recursive Formula: $t_1=54,\ t_n=\frac{1}{3}t_{n-1},\ n>1$

General Term: $t_n=ar^{n-1}$

$t_n=54\left(\frac{1}{3}\right)^{n-1}$

b. 18, 14, 10,… ← arithmetic $a=18$ $d=-4$

Recursive Formula: $t_1=18,\ t_n=t_{n-1}-4,\ n>1$

General Term: $t_n=a+(n-1)d$

$t_n=18+(n-1)(-4)$ → $t_n=18-4n+4$

$t_n=22-4n$

5. Determine the first 5 terms of the sequence defined by the following: $t_1=1,\ t_2=1,\ t_n=4t_{n-1}-t_{n-2},\ n\geq 3$

$t_n=4t_{n-1}-t_{n-2}$

$t_3=4t_2-t_1$

$t_3=4(1)-1$

$t_3=4-1$

$t_3=3$

$t_4=4t_3-t_2$

$=4(3)-1$

$=12-1$

$t_4=11$

$t_5=4t_4-t_3$

$=4(11)-(3)$

$=44-3$

$t_5=41$

∴ The first five terms are 1, 1, 3, 11, 41.

6. The sum of the first 20 terms of an arithmetic sequence is 730. The sum of the first 40 terms is 2660. What is the 50th term?

$S_n=\frac{n[2a+(n-1)d]}{2}$

set $n=20$ → $S_{20}=\frac{20[2a+19d]}{2}$

$730=10[2a+19d]$

① $730=20a+190d$

$S_n=\frac{n[2a+(n-1)d]}{2}$

$S_{40}=\frac{40[2a+39d]}{2}$

$2660=20[2a+39d]$

② $2660=40a+780d$

② $2660=40a+780d$

$-2$① $1460=40a+380d$

$\frac{1200}{400}=\frac{400d}{400}$

③ $d=3$

sub ③ into ①

$730=20a+190(3)$

$730=20a+570$

$\frac{20a}{20}=\frac{160}{20}$ → $a=8$

Additional Practice: pg 468 #3ac, 4, 7ace, 8, 9ac, 14ac, 15acd, 18ac, 19ac, 22ac, 23cd

continued on back

$a = 8 \qquad d = 3$

$$t_n = a + (n-1)d$$
$$= 8 + (n-1)3$$
$$= 8 + 3n - 3$$

$$t_n = 5 + 3n$$

set $n = 50$

$$t_{50} = 5 + 3(50)$$

$$t_{50} = 155$$