**Recursion Formulas**

Recall: Terms in a sequence can be determined using a formula for the nth term.

For example, determine the first three terms given each general term formula:

a) tn = 3n + 3 b) tn = 2n - 1

 t1 = t1 =

 t2 = t2 =

 t3 = t3 =

The first three terms are \_\_\_, \_\_\_, \_\_\_. The first three terms are \_\_\_, \_\_\_, and \_\_\_\_.

These equations above are called \_\_\_\_\_\_\_\_\_\_ formulas. They can be used to determine any term in a sequence without knowing the previous term(s).

 ie; If tn = 3n + 3 and we want to know what the 12th term is then...

t12 =

 =

 =

Alternatively, we can define terms in a sequence with respect to one or more of the previous terms using what is called a recursion formula.

A recursion formula has 3 parts:

* the first term(s)
* an equation/rule
* a validity statement for n

**Example 1**

a) Determine the first three terms given the following recursion formula:

t1 = 6, tn = tn-1 + 3, n > 1

(first term) (equation) (validity statement for n)

Since we already know t1, we proceed to figure out t2 using the equation

tn = tn-1 + 3 tn = tn-1 + 3 Therefore, the first three terms

 are \_\_\_, \_\_\_, and \_\_\_; it is an arithmetic

set n = 2 set n = 3 sequence in this case.

t2 = t3 =

t2 = t3 = Notice the line t3 = t2 + 3. This suggests that

 t3 cannot be determined until we know \_\_\_.

t2 = t3 =

t2 = t3 =

b) Determine the first three terms given the following recursion formula:

t1 = 2, tn = 3tn-1 , n > 1

The validity statement n > 1 suggests that only values of n greater than 1 can be used for this formula. Consequently, the first term that can be determined by the recursion equation is the second term.

tn = 3tn-1 tn = 3tn-1 Therefore, the first three terms

 are \_\_\_, \_\_\_\_, and \_\_\_; it is a geometric

set n = 2 set n = 3 sequence in this case.

t2 = t3 =

t2 = t3 =

t2 = t3 =

t2 = t3 =

Recursion equations for arithmetic sequences are usually written in the form tn = tn-1 + d.

Recursion equations for geometric sequences are usually written in the form tn = rtn-1.

where

* d is the common difference.
* r is the common ratio.

In general, if tn represents some term in a sequence then

* tn-1 refers to the term before tn
* tn-2 refers to the term before tn-1; in other words tn-2 is two terms before tn.

**Example 2**

Determine the first five terms for the recursion formula: t1 = 1, t2 = 1, tn = tn-1 + tn-2, n>2

Since n>2 and we already know t1 and t2 we start by finding t3...

tn = tn-1 + tn-2 tn = tn-1 + tn-2 tn = tn-1 + tn-2 The sequence is 1, 1, 2, 3, 5,..

 This is called the Fibonacci

t3 = t4 =t5 = sequence.

t3 = t4 = t5 =

t3 = t4 = t5 =

t3 = t4 = t5 =

**Example 3**

Create a recursion formula for each of the following sequences.

a) 5, -20, 80, -320.... b) 2, -3, -8, -13, ...