

## Exponent Practice

Simplify each expression; express all remaining exponents as positives.

a)  $5^{-2}$

$$= \frac{1}{5^2}$$

$$= \frac{1}{25}$$

b)  $\left(\frac{x}{2}\right)^{-3}$

$$= \left(\frac{2}{x}\right)^3 = \frac{8}{x^3}$$

$$= \frac{2^3}{x^3}$$

c)  $(-2x)^4$

$$= (-2)^4 (x)^4$$

$$= 16x^4$$

d)  $4.683^0$

$$= 1$$

e)  $\frac{x^2 q^3 y^{-1}}{z^{-4} w^5}$

$$= \frac{x^2 q^3 z^4}{w^5 y}$$

f)  $\frac{x^5}{x^1}$

$$= x^4$$

g)  $-2^4$

$$= -16$$

h)  $(-2)^4$

$$= 16$$

## Rational Exponents

What does it mean to have a fraction as an exponent?

For example, what is meant by the expression  $4^{\frac{5}{2}}$ ?  $4^{2.5}$

Intuitively, it would make sense that this quantity ( $4^{2.5}$ ) would be somewhere between  $4^2$  and  $4^3$ ; that is,  $4^{2.5}$  must be between the values 16 and 64. When we enter  $4^{2.5}$  into the calculator, we get a value of 32 which is indeed between  $4^2$  and  $4^3$  (but not in the middle).

To evaluate  $4^{\frac{5}{2}}$  without using a calculator, we need to make use of two important mathematical rules:

1. The expression  $x^{\frac{1}{n}}$  means  $\sqrt[n]{x}$  or the  $n^{\text{th}}$  root of  $x$ .

$$\text{eg; } 8^{\frac{1}{3}} = \sqrt[3]{8} = 2 \quad \text{or} \quad 81^{\frac{1}{4}} = \sqrt[4]{81} = 3$$

2. All powers can be written as a power of a power;

$$\text{eg; } 3^6 \text{ can equivalently be written as } (3^2)^3 \text{ or } (3^3)^2$$

Using the above two rules we can evaluate  $4^{\frac{5}{2}}$  as follows:

$$\begin{aligned}
 & 4^{\frac{5}{2}} \\
 &= (4^{\frac{5}{2}})^{\frac{1}{2}} \\
 &= (1024)^{\frac{1}{2}} \\
 &= \sqrt[2]{1024} \\
 &= 32
 \end{aligned}$$

or

$$\begin{aligned}
 & 4^{\frac{5}{2}} \\
 &= (4^{\frac{1}{2}})^5 \quad \checkmark \\
 &= (\sqrt{4})^5 \\
 &= (2)^5 \\
 &= 32
 \end{aligned}$$

### Examples

Evaluate each expression.

a)  $8^{\frac{2}{3}}$

$$\begin{aligned}
 &= (\sqrt[3]{8})^2 \\
 &= (2)^2 \\
 &= 4
 \end{aligned}$$

b)  $81^{\frac{3}{4}}$

$$\begin{aligned}
 &= (\sqrt[4]{81})^3 \\
 &= (3)^3 \\
 &= 27
 \end{aligned}$$

c)  $25^{\frac{3}{2}}$

$$\begin{aligned}
 &= (\sqrt{25})^3 \\
 &= (5)^3 \\
 &= 125
 \end{aligned}$$

d)  $36^{1.5}$

$$\begin{aligned}
 &= 36^{3/2} \\
 &= (\sqrt{36})^3 \\
 &= (6)^3 \\
 &= 216
 \end{aligned}$$

e)  $27^{-\frac{2}{3}}$

$$\begin{aligned}
 &= \frac{1}{27^{2/3}} \\
 &= \frac{1}{(\sqrt[3]{27})^2} \\
 &= \frac{1}{(3)^2}
 \end{aligned}$$

}  $= \frac{1}{9}$

f)  $16^{-1.25}$

$$\begin{aligned}
 &= 16^{-5/4} \\
 &= \frac{1}{16^{5/4}} \\
 &= \frac{1}{(\sqrt[4]{16})^5}
 \end{aligned}$$

}  $= \frac{1}{(2)^5} = \frac{1}{32}$

Homework: pg 229 # 1-3, 4bc, 5acf, 8, 10, 13-15