Solution

* Expand to get to standard form first

Quadratic Functions – Practice #2

1. Express each quadratic in vertex form.

a) $y = x^2 - 6x + 17$

$y = (x^2 - 6x + 9 - 9) + 17$

$y = (x^2 - 6x + 9) - 9 + 17$

$y = (x-3)(x-3) + 8$

$y = (x-3)^2 + 8$

b) $y = 2x^2 + 20x + 47$

$y = 2(x^2 + 10x + 25 - 25) + 47$

$y = 2(x^2 + 10x + 25) - 50 + 47$

$y = 2(x+5)(x+5) - 3$

$y = 2(x+5)^2 - 3$

c) $y = -2(x-1)(x+3)$

$y = -2(x^2 + 3x - 1x - 3)$

$y = -2(x^2 + 2x - 3)$

$y = -2x^2 - 4x + 6$

$y = -2(x^2 + 2x + 1 - 1) + 6$

$y = -2(x^2 + 2x + 1) + 2 + 6$

$y = -2(x+1)(x+1) + 8$

$y = -2(x+1)^2 + 8$

Switch to vertex form first.

Range: $\{y \in \mathbb{R} \mid y \geq -12\}$

2. Determine the inverse of each function (algebraically).

a) $y = 2(x-1)^2 - 6$

$2(y-1)^2 - 6 = x$

$\frac{2(y-1)^2}{2} = \frac{x+6}{2}$

$\sqrt{(y-1)^2} = \pm\sqrt{\frac{1}{2}(x+6)}$

$y - 1 = \pm\sqrt{\frac{1}{2}(x+6)}$

$y = \pm\sqrt{\frac{1}{2}(x+6)} + 1$

b) $y = x^2 + 6x - 12$

$y = (x^2 + 6x + 9 - 9) - 12$

$y = (x^2 + 6x + 9) - 9 - 12$

$y = (x+3)(x+3) - 21$

$y = (x+3)^2 - 21$

$(y+3)^2 - 21 = x$

$\sqrt{(y+3)^2} = \pm\sqrt{x+21}$

$y + 3 = \pm\sqrt{x+21}$

$y = \pm\sqrt{x+21} - 3$

c) $y = 3\sqrt{x+4} - 12$

$3\sqrt{y+4} - 12 = x$

$\frac{3\sqrt{y+4}}{3} = \frac{x+12}{3}$

$(\sqrt{y+4})^2 = \left(\frac{1}{3}(x+12)\right)^2$

$y + 4 = \left[\frac{1}{2}(x+12)\right]^2$

$y = \left[\frac{1}{2}(x+12)\right]^2 - 4, \quad x \geq -12$

Very important restriction needs to be here.

3. Determine the number of real solutions for each quadratic equation.

a) $9x^2 - 6x + 1 = 0$

$b^2 - 4ac$

$= (-6)^2 - 4(9)(1)$

$= 36 - 36$

$= 0$

$\therefore$ One real root.

b) $x^2 = 5x + 10$

$x^2 - 5x - 10 = 0$

$b^2 - 4ac$

$= (-5)^2 - 4(1)(-10)$

$= 25 + 40$

$= 65$ (positive)

$\therefore$ Two real roots

c) $2x^2 + 5x + 7 = 0$

$b^2 - 4ac$

$= (5)^2 - 4(2)(7)$

$= 25 - 56$

$= -31$ (negative)

$\therefore$ No real roots.

4. a) Determine the family of quadratic functions that have x-intercepts at 1 and 5.

$y = a(x-x_1)(x-x_2)$

$y = a(x-1)(x-5)$

b) Determine the specific quadratic equation from part a) that goes through the point (2, 6).

$y = a(x-1)(x-5)$

sub in $(2,6)$

$6 = a(2-1)(2-5)$

$6 = a(1)(-3)$

$\frac{6}{-3} = \frac{-3a}{-3}$

$a = -2$

$y = -2(x-1)(x-5)$

5. Expand and simplify the following expressions; note final answers should be expressed in simplified mixed radical form.

a) $\sqrt{14}\sqrt{2}$

$= \sqrt{28}$

$= \sqrt{4}\sqrt{7}$

$= 2\sqrt{7}$

b) $\frac{\sqrt{250}}{\sqrt{5}}$

$= \sqrt{50}$

$= \sqrt{25}\sqrt{2}$

$= 5\sqrt{2}$

c) $\frac{\sqrt{75}-\sqrt{27}}{2}$

$= \frac{\sqrt{25}\sqrt{3} - \sqrt{9}\sqrt{3}}{2}$

$= \frac{5\sqrt{3} - 3\sqrt{3}}{2}$

$= \frac{\cancel{2}\sqrt{3}}{\cancel{2}}$

$= \sqrt{3}$

6. Determine the point(s) of intersection between the given line and parabola.

a) ① $y = 2x^2 + 6x - 8$

② $y = 2x - 2$

sub ① into ②

$2x^2+6x-8 = 2x-2$

$2x^2+4x-6 = 0$

$2(x^2+2x-3) = 0$

$2(x+3)(x-1) = 0$

$x = -3$ or $x = 1$

Case 1 $(x=-3)$

$y = 2(-3)-2$

$y = -6-2$

$y = -8$

∴ P.O.I. @ $(-3,-8)$

Case 2 $(x=1)$

$y = 2(1)-2$

$y = 0$

∴ P.O.I. @ $(1,0)$

b) ① $y = -x^2 + 4x - 3$

② $y = -2x + \cancel{4}8$

Error; should be +8

sub ② into ①

$-2x+8 = -x^2+4x-3$

$x^2-6x+11 = 0$

b^2-4ac

$= (-6)^2-4(1)(11)$

$= 36-44$

$= -8$

No solution

∴ No points of intersection

7. Create a quadratic equation that goes through the following three points:

$$(0, -3) \qquad (3, 6) \qquad (-1, -18)$$

We are given the y-int, so start with standard form

$y = ax^2 + bx + c$

$c = -3$

$y = ax^2 + bx - 3$

Sub in one of the other points $\rightarrow (3, 6)$

$6 = a(3)^2 + b(3) - 3$

$6 = 9a + 3b - 3$

① $9 = 9a + 3b$

Sub in the other/Final point $\rightarrow (-1, -18)$

$-18 = a(-1)^2 + b(-1) - 3$

$-18 = a - b - 3$

② $-15 = a - b$

① $9 = 9a + 3b$

② $-15 = a - b$

① $9 = 9a + 3b$

$3 \times$ ② $=$ ③ $-45 = 3a - 3b$

① + ③ $\frac{-36}{12} = \frac{12a}{12}$

④ $a = -3$

Sub ④ into ①

$9 = 9(-3) + 3b$

$9 = -27 + 3b$

$\frac{36}{3} = \frac{3b}{3}$

$b = 12$

$y = ax^2 + bx + c$

$\therefore y = -3x^2 + 12x - 3$

8. Consider the two functions below:

$$① \quad y = 2x^2 - 6x + k$$
$$② \quad y = -2x + 3$$

$\therefore$ one solution for x.

If k is a constant and graphs of these two equations intersect at a single point, what are the coordinates of this point?

Don't worry about 'k' initially.

Sub ① into ②

$2x^2 - 6x + k = -2x + 3$

$\underbrace{2x^2 - 4x + k - 3 = 0}$

'k' must be a value such that this quadratic will have one solution; $b^2 - 4ac = 0$

$a = 2$

$b = -4$

$c = k - 3$

$b^2 - 4ac = 0$

$(-4)^2 - 4(2)(k-3) = 0$

$16 - 8(k-3) = 0$

$16 - 8k + 24 = 0$

$\frac{-8k}{-8} = \frac{-40}{-8}$

$\boxed{k = 5}$

$2x^2 - 4x + k - 3 = 0$

$2x^2 - 4x + 5 - 3 = 0$

$2x^2 - 4x + 2 = 0$

$2(x^2 - 2x + 1) = 0$

$2(x-1)(x-1) = 0$

③ $x = 1$

sub ③ into ②

$y = -2(1) + 3$

$y = 1$

$\therefore$ The single point of intersection occurs @ $(1, 1)$