Solution

Quadratic Functions – Practice #1

1. Solve the following quadratic equations.

P(-10) } -5, 2
S(-3) }

a) $x^2 + 6 = 6x$

$x^2 - 6x + 6 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(6)}}{2(1)}$

$= \frac{6 \pm \sqrt{12}}{2}$

$= \frac{6 \pm \sqrt{4}\sqrt{3}}{2}$

$= \frac{6 \pm 2\sqrt{3}}{2}$

$= \frac{2(3 \pm \sqrt{3})}{2}$

$= 3 \pm \sqrt{3}$ ← Exact values

b) $2x^2 = 3x + 5$

$2x^2 - 3x - 5 = 0$

$2x^2 - 5x + 2x - 5 = 0$

$x(2x - 5) + 1(2x - 5) = 0$

$(x + 1)(2x - 5) = 0$

$x = -1$ or $x = \frac{5}{2}$

c) $5x^2 - 40x + 80 = 0$ * Error; missing = 0

$5(x^2 - 8x + 16) = 0$

$5(x - 4)(x - 4) = 0$

$x = 4$

2. Expand and simplify the following expressions; note final answers should be expressed in simplified mixed radical form.

a) $\sqrt{8}\sqrt{3}$

$= \sqrt{24}$

$= \sqrt{4}\sqrt{6}$

$= 2\sqrt{6}$

b) $\frac{\sqrt{54}}{\sqrt{3}}$

$= \sqrt{18}$

$= \sqrt{9}\sqrt{2}$

$= 3\sqrt{2}$

c) $\frac{\sqrt{6}\sqrt{10}}{\sqrt{3}}$

$= \frac{\sqrt{60}}{\sqrt{3}}$

$= \sqrt{20}$

$= \sqrt{4}\sqrt{5}$

$= 2\sqrt{5}$

3. Determine the point(s) of intersection between the given line and parabola.

a) $y = x^2 + 6x - 10$ * Error; missing ∅

① $y = 2x - 14$

Sub ① into ②

$x^2 + 6x - 10 = 2x - 14$

$x^2 + 4x + 4 = 0$

$(x + 2)(x + 2) = 0$

③ $x = -2$

Sub ③ into ②

$y = 2(-2) - 14$

$y = -4 - 14$

$y = -18$

P.O.I. @ (-2, -18)

b) $y = -2x^2 + 3x - 9$

① $y = x - 8$

Sub ② into ①

$x - 8 = -2x^2 + 3x - 9$

$2x^2 - 2x + 1 = 0$

$b^2 - 4ac$

$= (-2)^2 - 4(2)(1)$

$= 4 - 8$

$= -4$ ← negative; ∴ No sol^n for x

The line and parabola do not cross.

No P.O.I.

4. Determine the distance separating the two points of intersection for the graphs:

① $y = -x^2 + 5x - 4$

② $y = x^2 - 7x - 4$

Sub ② into ①

$x^2 - 7x - 4 = -x^2 + 5x - 4$

$2x^2 - 12x = 0$

$2x(x-6) = 0$

$x = 0$ or $x = 6$

Case 1 ($x = 0$)

Sub $x = 0$ into ②

$y = (0)^2 - 7(0) - 4$

$y = -4$

P.O.I. @ $(0, -4)$ (x_1, y_1)

Case 2 ($x = 6$)

Sub $x = 6$ into ②

$y = (6)^2 - 7(6) - 4$

$y = 36 - 42 - 4$

$y = -10$

P.O.I @ $(6, -10)$ (x_2, y_2)

Length of Line

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$d = \sqrt{(0-6)^2 + (-4-(-10))^2}$

$d = \sqrt{36 + 36}$

$d = \sqrt{72}$

$d = \sqrt{36}\sqrt{2}$ * Exact Value

$d = 6\sqrt{2}$

≈ 8.49

5. Determine the family of quadratic functions that have an x-intercept of 4 and 12.

$y = a(x - x_1)(x - x_2)$

$y = a(x-4)(x-12)$

6. a) Determine the family of quadratic functions that have a vertex at (4, -2).

$y = a(x-p)^2 + q$

$y = a(x-4)^2 - 2$

b) Determine the specific quadratic equation from part a) that goes through the point (3, -5).

$y = a(x-4)^2 - 2$

sub in $(3, -5)$

$-5 = a(3-4)^2 - 2$

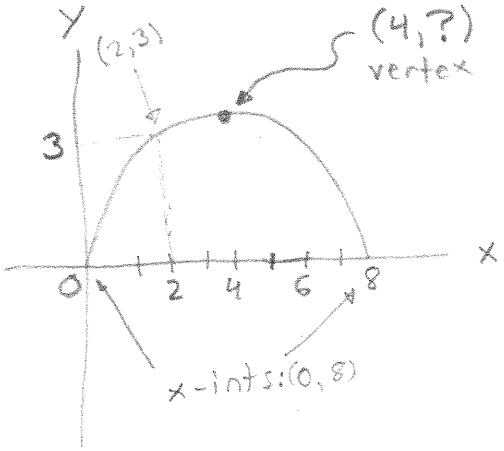
$-5 = a(1) - 2$

$-5 + 2 = a$

$a = -3$

$y = -3(x-4)^2 - 2$

7. A bridge is built such that the two ends that touch the ground are separated by 8 m. Two metres in from one side, the bridge is 3 m high. What is the highest point on the bridge?



$y = a(x-x_1)(x-x_2)$

$y = a(x-0)(x-8)$

sub in (2,3)

$3 = a(2-0)(2-8)$

$\frac{3}{-12} = \frac{-12a}{-12}$

$a = -\frac{1}{4}$

$\boxed{y = -\frac{1}{4}(x-0)(x-8)}$

Vertex

$x = \frac{0+8}{2}$

$x = 4$

$y = -\frac{1}{4}(4-0)(4-8)$

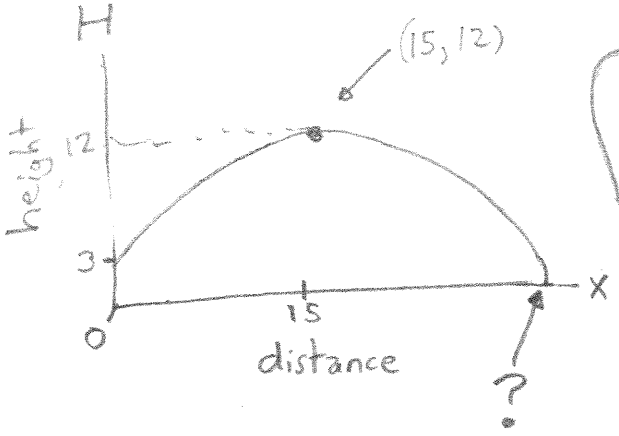
$y = -\frac{1}{4}(4)(-4)$

$y = 4$

vertex is (4,4)

∴ The highest point on the bridge is 4m @ the middle (when x=4).

8. A football is thrown in a parabolic arc down the field to a receiver. The football leaves the quarterback's hand from a height of 3 m above the ground. The football reaches a maximum height of 12 m once it has gone 15 m down the field. If the received misses the ball, how far down the field does it travel before hitting the ground?



$H = a(x-p)^2 + q$

$H = a(x-15)^2 + 12$

goes through (0,3)

$3 = a(0-15)^2 + 12$

$\frac{-9}{225} = \frac{225a}{225}$

$a = -0.04$

$H = -0.04(x-15)^2 + 12$

When the ball hits the ground, H = 0 so...

$0 = -0.04(x-15)^2 + 12$

$\frac{-12}{-0.04} = \frac{-0.04(x-15)^2}{-0.04}$

$\pm\sqrt{300} = \sqrt{(x-15)^2}$

$\pm 17.3 = x - 15$

$\pm 17.3 + 15 = x$

$x \approx 32.3$ or $x \approx -2.32$

inadmissable

∴ The football traveled 32.3m down the field before hitting the ground.