5.9 Exercises

A

1. Prove each identity.

   a) $\sin\theta = \tan\theta\cos\theta$

   b) $\cos\theta = \dfrac{\sin\theta}{\tan\theta}$

2. Explain how an identity is different from an equation.

3. Prove each identity.

   a) $\dfrac{1}{\sin\theta}(1+\sin\theta) = 1 + \dfrac{1}{\sin\theta}$

   b) $\sin\theta\left(1+\dfrac{1}{\sin\theta}\right) = 1+\sin\theta$

   c) $\cos\theta\left(\dfrac{1}{\cos\theta} - 1\right) = 1-\cos\theta$

   d) $\dfrac{\sin\theta}{\cos\theta\tan\theta} = 1$

B

4. **Knowledge/Understanding** Prove each identity.

   a) $\tan^2\theta = \dfrac{1-\cos^2\theta}{\cos^2\theta}$

   b) $\sin\theta\tan\theta = \dfrac{1-\cos^2\theta}{\cos\theta}$

   c) $\dfrac{\sin\theta}{1+\cos\theta} = \dfrac{1-\cos\theta}{\sin\theta}$

   d) $1-\tan\theta = -\tan\theta\left(1-\dfrac{1}{\tan\theta}\right)$

5. Prove each identity.

   a) $\sin\theta\tan\theta + \dfrac{1}{\cos\theta} = \dfrac{\sin^2\theta+1}{\cos\theta}$

   b) $\dfrac{1+\cos\theta}{1-\cos\theta} = \dfrac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}-1}$

   c) $\dfrac{1+\sin\theta}{1-\sin\theta} = \dfrac{\frac{1}{\sin\theta}+1}{\frac{1}{\sin\theta}-1}$

   d) $\left(\dfrac{1}{\cos\theta}+\tan\theta\right)(1-\sin\theta) = \cos\theta$

6. Prove each identity.

   a) $\dfrac{\sin^2\theta}{\tan^2\theta} = 1-\sin^2\theta$

   b) $\dfrac{1}{\sin^2\theta} - 1 = \dfrac{\cos^2\theta}{\sin^2\theta}$

   c) $\sin^2\theta = \dfrac{\tan^2\theta}{1+\tan^2\theta}$

   d) $\sin\theta + \dfrac{\cos\theta}{\tan\theta} = \dfrac{1}{\cos\theta\tan\theta}$

7. **Application**

   a) Prove this identity.
   $\sin\theta+\cos\theta = \sin\theta\cos\theta\left(\dfrac{1}{\sin\theta}+\dfrac{1}{\cos\theta}\right)$

   b) Predict a similar identity for the expression $\sin\theta+\tan\theta$, then prove it is correct.

   c) Establish another identity, similar to those in parts a and b.

8. a) Prove this identity.
   $\tan^2\theta = \left(\dfrac{1}{\cos\theta}-1\right)\left(\dfrac{1}{\cos\theta}+1\right)$

   b) Predict a similar identity for $\dfrac{1}{\tan^2\theta}$, then prove it is correct.

9. Prove each identity.

a) $(1 + \tan^2 \theta)(1 - \cos^2 \theta) = \tan^2 \theta$

b) $\left(1 + \frac{1}{\tan^2 \theta}\right)(1 - \sin^2 \theta) = \frac{1}{\tan^2 \theta}$

10. Prove each identity.

a) $\tan \theta + \frac{1}{\tan \theta} = \frac{1}{\sin \theta \cos \theta}$

b) $\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} = \frac{1}{\sin^2 \theta \cos^2 \theta}$

c) $\frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} = \left(\tan \theta + \frac{1}{\tan \theta}\right)^2$

d) $\sin^2 \theta = \cos \theta\left(\frac{1}{\cos \theta} - \cos \theta\right)$

11. **Communication** Prove the identity $\frac{\cos \theta}{1 + \sin \theta} + \frac{\cos \theta}{1 - \sin \theta} = \frac{2}{\cos \theta}$. Write to explain why the identity is not true for $\theta = \frac{\pi}{2}$ and $\frac{3\pi}{2}$.

12. a) Prove the identity $\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} = \frac{2}{\cos^2 \theta}$.

b) Establish a similar identity for $\frac{1}{1 + \cos \theta} + \frac{1}{1 - \cos \theta}$.

13. Prove each identity.

a) $\frac{1 + \sin \theta}{\cos \theta} = \frac{\cos \theta}{1 - \sin \theta}$

b) $\frac{1 + \sin \theta + \cos \theta}{1 - \sin \theta + \cos \theta} = \frac{1 + \sin \theta}{\cos \theta}$

14. Prove each identity.

a) $(\cos \theta - \sin \theta)^2 = 1 - 2 \sin \theta \cos \theta$

b) $\cos^4 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta$

c) $\frac{\tan^2 \theta}{1 + \tan^2 \theta} = \sin^2 \theta$

d) $\frac{\tan \theta + \cos \theta}{\sin \theta} = \frac{1}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$

15. **Thinking/Inquiry/Problem Solving** Determine an identity (different from any in these exercises) that involves all 3 trigonometric ratios.

a) Verify your identity numerically.

b) Prove your identity algebraically.

c) Write to explain how you found the identity.

C

16. Prove the identity $\frac{1 - \sin \theta}{1 + \sin \theta} = \tan \theta + \frac{1}{\cos \theta}$.