Homework: Worksheet # 1a, 3ad, 4ac, 5ad, 6ab, 8a, 10ac, 13a, 14ac

Proving Identities: Part 2

change everything to sine and cose

When proving identities, here are some strategies to consider:

1. Compare both sides of the equation. What is the same? What is different?

2. When proving an identity, it is often strategic to start manipulating the side that is busier.

3. All reciprocal trigonometric ratios ($\csc\theta$, $\sec\theta$, and $\cot\theta$) can easily be changed to one of the three primary trigonometric ratios ($\sin\theta$, $\cos\theta$, and $\tan\theta$).

4. It is usually recommended to change any instance of $\tan\theta$ or $\cot\theta$ to $\sin\theta$ and $\cos\theta$ using:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

5. The squared forms of sine and cosine can easily be changed to one another using.

rearranged versions of $\sin^2\theta+\cos^2\theta=1$ →

$$\sin^2\theta = 1-\cos^2\theta = (1-\cos\theta)(1+\cos\theta)$$
$$\cos^2\theta = 1-\sin^2\theta \quad (1-\sin\theta)(1+\sin\theta)$$

Furthermore, the right side of these identities can be factored as differences of squares.

6. The conjugate of a binomial is the same two-termed expression presented with the opposite operator in between terms. For example:

→ the conjugate of $A+B$ is $A-B$
→ the conjugate of $A-B$ is $A+B$

The conjugate can be used to complete a difference of squares. For example;

$$\frac{\cos\theta}{1+\sin\theta}\cdot\frac{(1-\sin\theta)}{(1-\sin\theta)}$$

$$(1+\sin\theta)(1-\sin\theta) = 1-\cancel{\sin\theta}+\cancel{\sin\theta}-\sin^2\theta = 1-\sin^2\theta$$

$$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cancel{\cos\theta}(1-\sin\theta)}{\cos^{\cancel{2}}\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

Examples

Prove the following:

a) $\frac{\tan\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta\cos\theta}$

$L.S. = \frac{\tan\theta}{(1-\cos\theta)}\frac{(1+\cos\theta)}{(1+\cos\theta)}$ $\quad R.S. = \frac{1+\cos\theta}{\sin\theta\cos\theta}$

$= \frac{\tan\theta(1+\cos\theta)}{1-\cos^2\theta}$

$= \frac{\tan\theta\,(1+\cos\theta)}{\sin^2\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}\,\frac{(1+\cos\theta)}{1}}{\sin^2\theta}$

$= \frac{\cancel{\sin\theta}\,(1+\cos\theta)}{\cos\theta} \cdot \frac{1}{\sin^{\cancel{2}}\theta}$

(divide by $\sin^2\theta$ is same as multiply by $\frac{1}{\sin^2\theta}$)

$= \frac{1+\cos\theta}{\sin\theta\cos\theta}$

$L.S. = R.S.$

QED

b) $\frac{2\sin\theta - \cos^2\theta - 2}{\sin\theta + 3} = \sin\theta - 1$

$L.S. = \frac{2\sin\theta - \cos^2\theta - 2}{\sin\theta + 3}$ $\quad R.S. = \sin\theta - 1$

$= \frac{2\sin\theta - (1-\sin^2\theta) - 2}{\sin\theta + 3}$

$= \frac{2\sin\theta - 1 + \sin^2\theta - 2}{\sin\theta + 3}$

$= \frac{(\sin^2\theta + 2\sin\theta - 3)}{\sin\theta + 3}$

Think $x^2 + 2x - 3$
$= (x+3)(x-1)$

$= \frac{\cancel{(\sin\theta+3)}(\sin\theta - 1)}{\cancel{(\sin\theta + 3)}}$

$= \sin\theta - 1$

$L.S. = R.S.$

QED

c) $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$L.S. = \tan\theta + \cot\theta$ $\quad R.S. = \sec\theta\csc\theta$

$= \frac{\sin\theta}{\sin\theta}\left(\frac{\sin\theta}{\cos\theta}\right) + \left(\frac{\cos\theta}{\sin\theta}\right)\frac{\cos\theta}{\cos\theta}$ $\quad = \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ $\quad = \frac{1}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta}$

$L.S. = R.S.$

QED

d) $\sin\theta + \sin\theta\cot^2\theta = \csc\theta$

$L.S. = \sin\theta + \sin\theta\cot^2\theta$ $\quad R.S. = \csc\theta$

$= \sin\theta + \sin\theta\left(\frac{\cos\theta}{\sin\theta}\right)^2$ $\quad = \frac{1}{\sin\theta}$

$= \sin\theta + \frac{\cancel{\sin\theta}}{1}\frac{\cos^2\theta}{\sin^{\cancel{2}}\theta}$

$= \frac{\sin\theta}{\sin\theta}\frac{\sin\theta}{1} + \frac{\cos^2\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$

$L.S. = R.S.$

QED