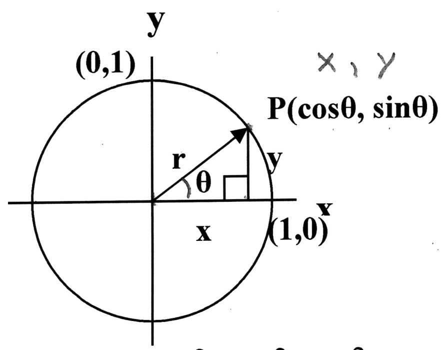
Homework: pg 310 #1ab, 2, 3, 4 (cross multiply), 5, 7ac, 8bcdf, 11

<u>Proving Trigonometric Identities</u> : Part 1

Recall: The Unit Circle



The Pythagorean Theorem states that $a^2 + b^2 = c^2$.

If we consider the right triangle in the above diagram, we get:

$$y^2 + x^2 = r^2$$

But for the unit circle when r = 1,

$\cos\theta = \frac{x}{r}$

$= \frac{x}{1}$ ← $r=1$

$\boxed{\cos\theta = x}$

$$\boxed{\begin{array}{l} x = \cos\theta \\ y = \sin\theta \end{array}}$$

So, we substitute these values into the equation above to get:

SYR CXR TYX

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

L.S. $= \sin^2\theta + \cos^2\theta$

$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{y^2 + x^2}{r^2}$

$= \frac{r^2}{r^2}$

$= 1$

This identity is referred to as a <u>*Pythagorean Identity.*</u>

This can also be rearranged to get the following variations:

$$\sin^2\theta = 1 - \cos^2\theta \quad ✓$$
$$\cos^2\theta = 1 - \sin^2\theta \quad ✓$$

Also, it may have been briefly mentioned before that:

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

R.S. $= \frac{\sin\theta}{\cos\theta}$

$= \frac{y/r}{x/r}$

$= \frac{y}{r} \div \frac{x}{r}$

$= \frac{y}{r} \cdot \frac{r}{x}$

$= \frac{y}{x}$

$= \tan\theta$

This identity is referred to as a <u>*Quotient Identity.*</u>

A few other identities that may be helpful include the following:

$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

other Pythagorean Identities

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Examples

Prove the following trig identities:

* (all angles changed to θ) *

a) $\dfrac{\sin x}{\tan x} = \cos x$

L.S. $= \dfrac{\sin x}{\tan x}$ R.S. $= \cos x$

$= \dfrac{\sin x}{\sin x / \cos x}$

$= \dfrac{\sin x}{1} \div \dfrac{\sin x}{\cos x}$

$= \dfrac{\cancel{\sin x}}{1} \cdot \dfrac{\cos x}{\cancel{\sin x}}$

$= \cos x$

L.S. = R.S.

QED

b) $\dfrac{1}{\cos x} - \cos x = \sin x \tan x$

$\sin^2 x = 1 - \cos^2 x$

L.S. $= \dfrac{1}{\cos x} - \dfrac{\cos x}{1}$ R.S. $= \sin x \tan x$

$= \dfrac{1}{\cos x} - \dfrac{\cos^2 x}{\cos x}$

$= \dfrac{1 - \cos^2 x}{\cos x}$

$= \dfrac{\sin^2 x}{\cos x}$

R.S. $= \dfrac{\sin x}{1} \cdot \dfrac{\sin x}{\cos x}$

$= \dfrac{\sin^2 x}{\cos x}$

L.S. = R.S.

QED

c) $2\sin^2 x - 1 = \sin^2 x - \cos^2 x$

L.S. $= 2\sin^2 x - 1$ R.S. $= \sin^2 x - \cos^2 x$

$= \sin^2 x - (1 - \sin^2 x)$

$= \sin^2 x - 1 + \sin^2 x$

$= 2\sin^2 x - 1$

L.S. = R.S.

QED

d) $\dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x} = \dfrac{1}{\sin^2 x \cos^2 x}$

L.S. $= \dfrac{\cos^2 x}{\cos^2 x}\dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x}\dfrac{\sin^2 x}{\sin^2 x}$ R.S. $= \dfrac{1}{\sin^2 x \cos^2 x}$

$= \dfrac{\cos^2 x}{\sin^2 x \cos^2 x} + \dfrac{\sin^2 x}{\sin^2 x \cos^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$

$= \dfrac{1}{\sin^2 x \cos^2 x}$

L.S. = R.S.

QED

e) $\dfrac{\sin^2 x}{1 - \cos x} = 1 + \cos x$

L.S. $= \dfrac{\sin^2 x}{1 - \cos x}$ R.S. $= 1 + \cos x$

$= \dfrac{1 - \cos^2 x}{1 - \cos x}$

$= \dfrac{\cancel{(1 - \cos x)}(1 + \cos x)}{\cancel{(1 - \cos x)}}$

$= 1 + \cos x$

L.S. = R.S.

QED

f) $\tan\theta = \dfrac{\sin\theta + \sin^2\theta}{\cos\theta(1 + \sin\theta)}$

$x + x^2 = x(1 + x)$

L.S. $= \tan\theta$ R.S. $= \dfrac{\sin\theta + \sin^2\theta}{\cos\theta(1 + \sin\theta)}$

$= \dfrac{\sin\theta\,\cancel{(1 + \sin\theta)}}{\cos\theta\,\cancel{(1 + \sin\theta)}}$

$= \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

L.S. = R.S.

QED