Homework: pg 520 # 3, 4, 5, 6, 7, 9, (15)

Present Value Annuities

A present value annuity is similar to a future value annuity with some noticeable differences:

- The large lump sum of money is presently available.
- Regular withdrawals are made from the account (lump sum) until the balance is zero.

Both future value and present value annuities collect interest while the deposits or withdrawals occur.

One of the most common applications of a present value annuity is to model the financial terms related to spending a retirement fund.

interest

withdrawals

Example 1

Suppose a large sum of money resides in an account that is earning 8%/a interest compounded annually. Regular withdrawals of \$5000 are to be made each year for 5 years at which time the account will then be reduced to zero. What must have been the size of the original lump sum for this arrangement to work?

Time	1	2	3	4	5
	5000	5000	5000	5000	5000

$\frac{5000}{1.08}$

$\frac{5000}{(1.08)^2}$

$\frac{5000}{(1.08)^3}$

$\frac{5000}{(1.08)^4}$

$\frac{5000}{(1.08)^5}$

$$P = \underbrace{\frac{5000}{(1.08)^5} + \frac{5000}{(1.08)^4} + \frac{5000}{(1.08)^3} + \frac{5000}{(1.08)^2} + \frac{5000}{(1.08)^1}}_{\text{geometric series}} \rightarrow P = a\frac{(r^n - 1)}{r - 1}$$

(Initial Lump Sum)

$a = \frac{5000}{(1.08)^5}$

$r = 1.08$

$n = 5$

$$= \frac{5000}{(1.08)^5} \times \frac{(1.08^5 - 1)}{1.08 - 1}$$

$$= \frac{5000(1.08^5 - 1)}{(1.08)^5(0.08)}$$

$$= \$19963.55$$

Use the result in the previous example to create an equation for a present value annuity.

$$P = \frac{5000(1.08^5 - 1)}{(1.08)^5(0.08)}$$

$$= \frac{R[(1+i)^n - 1]}{(1+i)^n i}$$

$$= \frac{R}{i} \cdot \frac{[(1+i)^n - 1]}{(1+i)^n}$$

$$= \frac{R}{i} \cdot \left[\frac{(1+i)^n}{(1+i)^n} - \frac{1}{(1+i)^n}\right]$$

$x^{-n} = \frac{1}{x^n}$

$$= \frac{R}{i} \cdot [1 - (1+i)^{-n}]$$

$$\boxed{P = \frac{R[1-(1+i)^{-n}]}{i}}$$

Present Value (lump sum)

Regular Withdrawals

interest rate

# of withdrawals / # of times interest collected

Example 2

How much money needs to be in a retirement fund in order to be able to withdraw $1500 a month for 25 years if the account is collecting 6%/a interest compounded monthly?

P = ?

R = 1500

i = $0.06 \div 12 = 0.005$

n = $25 \times 12 = 300$

$$P = \frac{R[1-(1+i)^{-n}]}{i}$$

$$= \frac{1500[1-(1.005)^{-300}]}{0.005}$$

$$= \$232\,810.30$$

Create an equation for the regular withdrawal amounts by isolating the withdrawal amount, R, in the present value annuity equation.

$$\frac{P}{1} = \frac{R[1-(1+i)^{-n}]}{i}$$

$$\frac{R[\cancel{1-(1+i)^{-n}}]}{\cancel{1-(1+i)^{-n}}} = \frac{Pi}{1-(1+i)^{-n}}$$

$$\boxed{R = \frac{Pi}{1-(1+i)^{-n}}}$$

Example 3

Anika has saved up \$1000000 for her retirement. She would like to make monthly withdrawals from the account for 30 years. If the interest rate is predicted to be 4%/a compounded monthly over the duration of the annuity, how much will she be allowed to withdraw each month?

P = 1000000

R = ?

i = $0.04 \div 12 = 0.00\overline{3}$

n = $30 \times 12 = 360$

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

$$= \frac{1000000(0.00\overline{3})}{1-(1.00\overline{3})^{-360}}$$

$$= \$4774.15/\text{month}$$