

Power of a Product
 $(AB)^n = A^n B^n$

Homework: pg 167 #1-7ace, 12, (13), 14

Operations with Radicals

Power of Quotient
 $\left(\frac{A}{B}\right)^n = \frac{A^n}{B^n}$

Minds-On

Expand the following:

a) $(3x)^2$
 $= 3^2 x^2$

b) $\left(\frac{x}{3}\right)^4$
 $= \frac{x^4}{3^4}$

The first example uses the Power of a Product Law while the second example uses the Power of a Quotient Law. Note, while it is unconventional, both laws can be applied in reverse order. For example, change the following...

a) $3^2 x^2$
 $= (3x)^2$

b) $\frac{x^4}{3^4}$
 $= \left(\frac{x}{3}\right)^4$

The square root function is actually a power with an exponent equal to $\frac{1}{2}$.

Example 1

Evaluate the following using an exponent:

a) $\sqrt{9}$
 $= 9^{1/2}$
 $= 3$

b) $\sqrt{121}$
 $= 121^{1/2}$
 $= 11$

c) $\sqrt{64}$
 $= 64^{1/2}$
 $= 8$

Example 2

If we apply the power of a product and power of a quotient laws in reverse order, we may simplify the following:

$$\begin{aligned} \text{a) } & \sqrt{3} \times \sqrt{2} \\ & = 3^{1/2} \times 2^{1/2} \\ & = (3 \times 2)^{1/2} \\ & = \sqrt{3 \times 2} \\ & = \sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{\sqrt{5}}{\sqrt{3}} \\ & = \frac{5^{1/2}}{3^{1/2}} \\ & = \left(\frac{5}{3}\right)^{1/2} \\ & = \sqrt{\frac{5}{3}} \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{\sqrt{x}}{\sqrt{2}} \cdot \frac{\sqrt{3}}{1} \\ & = \frac{\sqrt{x} \sqrt{3}}{\sqrt{2}} \\ & = \frac{\sqrt{3x}}{\sqrt{2}} \\ & = \sqrt{\frac{3x}{2}} \end{aligned}$$

In general, we may summarize to create the following rules

$$\boxed{\sqrt{A} \times \sqrt{B} = \sqrt{AB}}$$

and

$$\boxed{\frac{\sqrt{A}}{\sqrt{B}} = \sqrt{\frac{A}{B}}}$$

The product rule can be used to manipulate expressions that contain radicals so that they are presented in mixed radical form.

Example 3

Manipulate each of the following expressions to present them in mixed radical form.

$$\begin{aligned} \text{a) } & \sqrt{12} \\ & = \sqrt{4} \sqrt{3} \\ & = 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{b) } & \sqrt{32} \\ & = \sqrt{16} \sqrt{2} \\ & = 4\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{c) } & \sqrt{75} \\ & = \sqrt{25} \sqrt{3} \\ & = 5\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{d) } & \sqrt{50} - \sqrt{8} \\ & = \sqrt{25} \sqrt{2} - \sqrt{4} \sqrt{2} \\ & = 5\sqrt{2} - 2\sqrt{2} \\ & = 3\sqrt{2} \end{aligned}$$

$5x^{-2}x$