Multiplying and Dividing Rational Expressions

Multiplication

Recall: $\frac{2}{3} \cdot \frac{6}{5}$

$= \frac{12}{15}$

$= \frac{4}{5}$

$\therefore \frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$ **where** $B \neq 0$, $D \neq 0$

Example 1

Simplify and state all restrictions.

$\frac{\cancel{x}\cancel{x}\cancel{x}x}{\cancel{x}\cancel{x}\cancel{x}}$

a) $\frac{\cancel{7}^{1}}{2\cancel{x^3}} \cdot \frac{\cancel{x^4}^{1}}{\cancel{14}^{2}}$

$= \frac{x}{4}$ or $= \frac{7x^4}{28x^3}$

$= \frac{1x^1}{4}$

$x \neq 0$

b) $\frac{3m+12}{5m} \cdot \frac{6m^3}{2m+8}$

$= \frac{3\cancel{(m+4)}}{5\cancel{m}} \cdot \frac{\cancel{6}^{3}\cancel{m^3}^{2}}{\cancel{2}^{1}\cancel{(m+4)}}$

$= \frac{9m^2}{5}$

$m \neq 0, -4$

c) $\frac{6n^2}{n+3} \cdot \frac{5n+15}{8n^3}$

$= \frac{3\cancel{6}\cancel{n^2}}{\cancel{n+3}} \cdot \frac{5\cancel{(n+3)}}{4\cancel{8}\cancel{n^3}^{1}}$

$= \frac{15}{4n}$

$n \neq -3, 0$

d) $\frac{x^2+3xy}{x^2-xy-42y^2} \cdot \frac{x^2-10xy+21y^2}{x^2-9y^2}$

$= \frac{x\cancel{(x+3y)}}{\cancel{(x-7y)}(x+6y)} \cdot \frac{\cancel{(x-3y)}\cancel{(x-7y)}}{\cancel{(x-3y)}\cancel{(x+3y)}}$

$= \frac{x}{x+6y}$

$x-7y \neq 0$

$x \neq 7y$, $x \neq -6y$, $x \neq \pm 3y$

Division

Recall: $\frac{1}{2} \div \frac{5}{4}$ copy flip flip

$= \frac{1}{2} \times \frac{4}{5}$

$= \frac{4}{10} = \frac{2}{5}$

$\therefore \boxed{\frac{A}{B} \div \frac{C}{D}}$ restrictions

$$= \frac{\frac{A}{B}}{\frac{C}{D}}$$

if $C = 0$: $\frac{\frac{A}{B}}{\frac{0}{D}} = \frac{\left(\frac{A}{B}\right)}{0}$ dividing by zero

$$= \frac{A}{B} \times \frac{D}{C}$$

$$= \frac{AD}{BC}$$

where $B \neq 0$, $C \neq 0$, $D \neq 0$

Example 2

Simplify and state all restrictions.

a) $\frac{y^3}{6} \div \frac{y^2}{3y-6}$

$= \frac{y^3}{6} \cdot \frac{3y-6}{y^2}$

$= \frac{\cancel{y^3}^{1}}{\cancel{6}\,2} \cdot \frac{\cancel{3}^{1}(y-2)}{\cancel{y^2}}$

$= \frac{y(y-2)}{2}$

$y \neq 2, 0$

b) $\frac{x^2+5x+6}{2x+4} \div \frac{x^2-9}{x}$ copy flip flip

$= \frac{\cancel{(x+2)}\cancel{(x+3)}}{2\cancel{(x+2)}} \cdot \frac{x}{(x-3)\cancel{(x+3)}}$

$= \frac{x}{2(x-3)}$

$x \neq -2, 0, \pm 3$

Homework: Pg 121 #1-3, 6ab, 7ab, 9, 10, (13)