

Homework – Complete worksheet

Mortgages: Part 2

Example 1

Consider the scenario from yesterday where a 25 year mortgage for \$220,000 at a quoted interest rate cost \$1041.14/month. What would be the monthly savings and total savings, if Sally had shopped around and found a 25 year mortgage with an interest rate of 2.75%/a instead of 3%/a?

$$\begin{aligned} \text{Semi-annual rate} &= \frac{0.0275}{2} = 0.01375 \\ \sqrt[6]{(1+i)^6} &= \sqrt[6]{1.01375} \\ 1+i &= 1.002278647 \\ i &= 0.002278647 \end{aligned}$$

$$\begin{aligned} P &= 220000 \\ R &= ? \\ i &= 0.002278647 \\ n &= 25 \times 12 = 300 \end{aligned}$$

$$\begin{aligned} R &= \frac{P \cdot i}{[1 - (1+i)^{-n}]} \\ &= \frac{220000(0.002278647)}{[1 - (1.002278647)^{-300}]} \\ &= \$1013.13/\text{month} \end{aligned}$$

$$\begin{aligned} \text{Monthly Savings} &= 1041.14 - 1013.13 \\ &= \$28.01/\text{month} \end{aligned}$$

$$\begin{aligned} \text{Total Savings} &= 300 \times 28.01 \\ &= \$8403 \end{aligned}$$

Example 2

Ashlyn is purchasing a house for \$250,000 with no downpayment.

Consider two mortgage plans:

Plan 1

→ 8%/a for the first 10 years

→ 6%/a for the final 10 years

Plan 2

→ 6%/a for the first 10 years

→ 8%/a for the final 10 years

Which plan is better?

$$\begin{aligned} \frac{8\%/a}{\sqrt[6]{(1+i)^6} = \sqrt[6]{1.04}} \\ i = 0.006558197 \end{aligned}$$

$$\begin{aligned} \frac{6\%/a}{\sqrt[6]{(1+i)^6} = \sqrt[6]{1.03}} \\ i = 0.004938622 \end{aligned}$$

Plan 1

$$R = \frac{P_i}{1 - (1+i)^{-n}}$$

$$= \frac{250000(0.006558197)}{1 - (1.006558197)^{-240}}$$

$$= \$2070.89/\text{month}$$

$$P = \frac{R[1 - (1+i)^{-n}]}{i}$$

$$= \frac{2070.89[1 - (1.006558197)^{-120}]}{0.006558197}$$

$$= \$171657.37$$

$$R = \frac{P_i}{1 - (1+i)^{-n}}$$

$$= \frac{171657.37(0.004938622)}{1 - (1.004938622)^{-120}}$$

$$= \$1899.41/\text{month}$$

Conclusion

$$\text{total} = 120(2070.89) + 120(1899.41)$$

$$= \$476436$$

Plan 2

$$R = \frac{P_i}{1 - (1+i)^{-n}}$$

$$= \frac{250000(0.004938622)}{1 - (1.004938622)^{-240}}$$

$$= \$1780.47/\text{month}$$

$$P = \frac{R[1 - (1+i)^{-n}]}{i}$$

$$= \frac{1780.47[1 - 1.004938622^{-120}]}{0.004938622}$$

$$= \$160908.64$$

$$R = \frac{P_i}{1 - (1+i)^{-n}}$$

$$= \frac{160908.64(0.006558197)}{1 - (1.006558197)^{-120}}$$

$$= \$1941.22/\text{month}$$

$$\text{total} = 120(1780.47) + 120(1941.22)$$

$$= \$446603$$

Conclusion

→ It is most important to negotiate a low interest rate on your mortgage for the earliest part of your amortization period.