

Mortgages (Part 1)

A mortgage is a type of loan used to purchase an accommodation (such as a house, cottage, condominium, etc...). The duration over which the loan is repaid is called the amortization period.

A mortgage is an example of a present value annuity. Typically, a bank or credit union might be the investor. They provide you with a large sum of money to purchase a home and then they receive regular payments from you until the loan is paid off. Meanwhile, interest continues to be charged on the loan until the balance is zero.

In terms of a mortgage...

- P is the amount borrowed for the mortgage.
- R is the monthly payments.
- i is the interest rate.
- n is the number of mortgage payments or interest charges.

In Canadian law states that all quoted interest rates for mortgages must be compounded semi-annually.

Example 1

Determine the equivalent monthly interest rate if the quoted mortgage rate is 12%/a.

Semi-annual interest rate is $\frac{12\%}{2} = 6\%$ every 6 months

$$\sqrt[6]{(1+i)^6} = \sqrt[6]{1.06}$$

$$1+i = 1.009758794$$

$$i = 0.009758794 \leftarrow \text{monthly interest rate}$$

Example 2

Sally Giroux would like to purchase a house for \$300,000. She currently has funds to make a downpayment of \$80,000. She goes to the bank and is offered a 25 year mortgage at an interest rate of 3%/a.

a) What is the equivalent monthly interest rate?

$$\begin{aligned}\text{Semi-annual rate} &= \frac{3\%}{2} = 1.5\% \\ \sqrt[6]{(1+i)^6} &= \sqrt[6]{1.015} \\ 1+i &= 1.002484517 \\ i &= 0.002484517\end{aligned}$$

b) What will be her monthly payments?

$$\begin{aligned}P &= 300000 - 80000 = 220000 \\ R &= ? \\ i &= 0.002484517 \\ n &= 25 \times 12 = 300\end{aligned}$$

$$\begin{aligned}R &= \frac{Pi}{1 - (1+i)^{-n}} \\ &= \frac{(220000)(0.002484517)}{1 - (1.002484517)^{-300}} \\ &= \$1041.14/\text{month}\end{aligned}$$

c) How much was still owing on the house after 5 years? (20 years remain)

$$\begin{aligned}P &= ? \\ R &= 1041.14 \\ i &= 0.002484517 \\ n &= 20 \times 12 = 240\end{aligned}$$

$$\begin{aligned}P &= \frac{R[1 - (1+i)^{-n}]}{i} \\ &= \frac{1041.14[1 - (1.002484517)^{-240}]}{0.002484517} \\ &= \$188044.11\end{aligned}$$

d) Over the first five years, how much was paid towards the interest and how much was paid to the principal (reducing the amount owing on the loan).

$$\begin{aligned}\text{Principal} &= 220000 - 188044.11 = \$31955.89 \\ \text{Interest} &= \text{total payments} - \text{principal paid} \\ &= (1041.14)(5 \times 12) - 31955.89 \\ &= \$30512.51\end{aligned}$$