

Modeling Periodic Behaviour

Periodic behaviour describes a function that repeats.

Some examples of periodic behaviour are:

1. The outside temperature as a function of time in years.
2. The amount of daylight on a daily basis.
3. The electrical activity around the heart as viewed on an electrocardiogram (ECG).
4. The population of tent caterpillars; pattern repeats every 10 years.
5. The voltage of an AC (alternating current) electric circuit.
6. Financial stability in the market; recessions occur about once every 10 years; 1981, 1991, 2001, 2009

Periodic events are often discussed using the following key terms:

- Cycle – one repetition in a repeating process.
- Period (T) – The change in the independent variable (typically time in seconds) corresponding to one cycle. *(must be positive)*
- Frequency (f) – the number of cycles that occur per unit of the independent variable. If the independent variable is time measured in seconds then the frequency is measured in s^{-1} or Hz (Hertz). *(must be positive)*
- Line of Equilibrium (simply called the “axis” in the textbook) – a horizontal line that is positioned halfway up from the bottom of the graph to the top.
- Amplitude – Half of the graph’s range or the vertical distance from the line of equilibrium to the top of the graph. *(must be positive)*

Example 1

If the maximum temperature in Guelph is 28°C and the minimum temperature is -12°C each year then calculate the amplitude and the line of equilibrium:

$$\text{Amplitude} = \frac{\text{maximum} - \text{minimum}}{2} = \frac{(28^{\circ}\text{C}) - (-12^{\circ}\text{C})}{2} = \frac{40^{\circ}\text{C}}{2} = 20^{\circ}\text{C}$$

$$\text{Line of Equilibrium} = \frac{\text{maximum} + \text{minimum}}{2} = \frac{(28^{\circ}\text{C}) + (-12^{\circ}\text{C})}{2} = \frac{16^{\circ}\text{C}}{2} = 8^{\circ}\text{C}$$

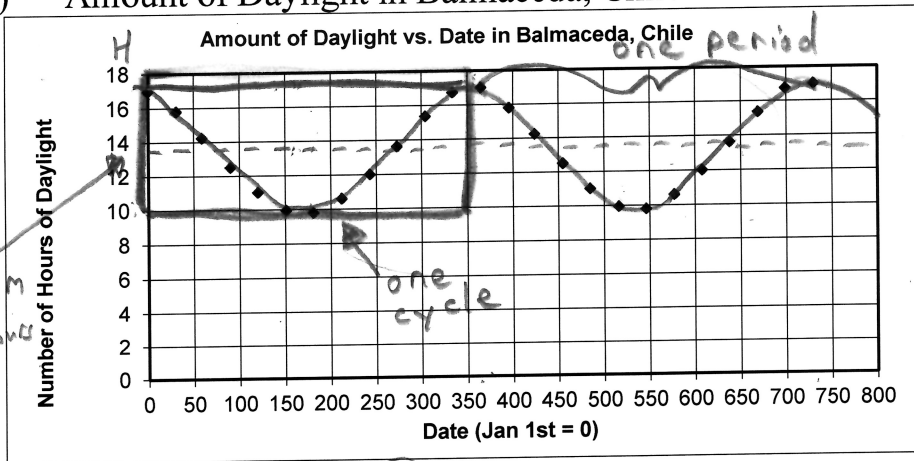
Period and frequency are related by the equations:

$$T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T}$$

Example 2

Determine if each graph describes periodic behaviour. If the graph is periodic then label one cycle, determine the amplitude, period and frequency. + eqⁿ of the line of equilibrium

a) Amount of Daylight in Balmaceda, Chile



$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2} = \frac{17 - 10}{2} = 3.5 \text{ hours}$$

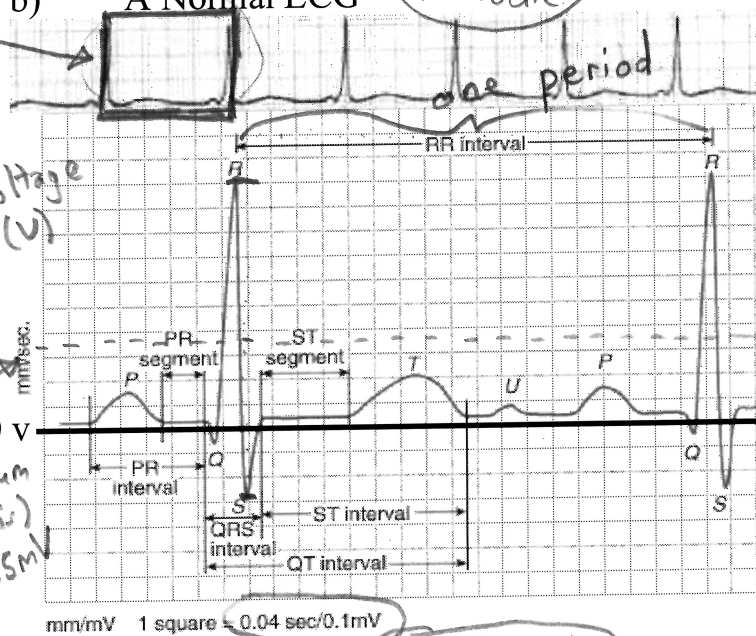
$$\text{Period} = 365 \text{ days}$$

$$\text{Frequency} = \frac{1}{T} = \frac{1}{365 \text{ days}} = \frac{1}{365} \text{ days}^{-1}$$

Equation of the line of equilibrium \rightarrow

$$H = \frac{\text{max} + \text{min}}{2} = \frac{17 + 10}{2} = 13.5 \text{ hours}$$

b) A Normal ECG



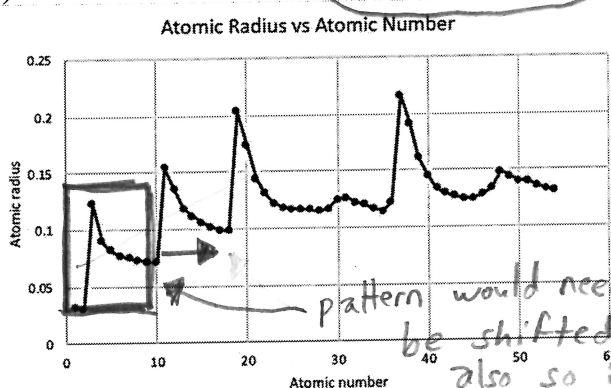
$$\text{Amplitude} = 6.5 \times 0.1 \text{ mV} = 0.65 \text{ mV}$$

$$\text{Period} = 19 \times 0.04 \text{ s} = 0.76 \text{ s}$$

$$\text{Frequency} = \frac{1}{T} = \frac{1}{0.76} \approx 1.32 \text{ Hz}$$

Equation of the line of equilibrium $\rightarrow V = 3.5 \times 0.1 \text{ mV} = 0.35 \text{ mV}$

c) Radius of Atom **Not Periodic**



Amplitude =

Period =

Frequency =

Equation of the line of equilibrium \rightarrow

Not Applicable; graph is not periodic