Homework: pg 153 # 1-3, 4bcd, 8, 9, 11, 12, 13

Maximum/Minimum Values of a Quadratic Function

The vertex (maximum/minimum) of a quadratic function can be determined using two methods:

1. Completing the square
2. Using x-intercepts

Example 1

Determine the maximum/minimum value of each quadratic relationship by completing the square:

a) $y = x^2 - 4x + 9$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = 4$

$y = (x^2 - 4x + 4 - 4) + 9$

$y = (x^2 - 4x + 4) + 9 - 4$

$y = (x-2)(x-2) + 5$

$y = (x-2)^2 + 5$

$\frac{b}{2} = \frac{-4}{2} = -2$

The minimum occurs @ (2, 5)

b) $y = -3x^2 + 12x - 8$

$y = -3(x^2 + 4x) - 8$ $\times(-3)$

$\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 4$

$y = -3(x^2 + 4x + 4 - 4) - 8$

$y = -3(x^2 + 4x + 4) - 8 + 12$

$y = -3(x+2)(x+2) + 4$

$y = -3(x+2)^2 + 4$

$\frac{b}{2} = \frac{4}{2} = 2$

The maximum occurs @ (-2, 4)

Example 2

Determine the maximum/minimum value of each quadratic relationship by using the x-intercepts:

a) $y = x^2 - 4x + 3$

$y = (x-1)(x-3)$

x-ints: 1 & 3

Vertex

$x = \frac{1+3}{2}$

$x = 2$

$y = (2-1)(2-3)$

$= (1)(-1)$

$y = -1$

The minimum occurs @ (2, -1).

b) $y = x^2 - 2x + 4$

$b^2 - 4ac$

$= (-2)^2 - 4(1)(4)$

$= -12$

Not factorable!

No x-ints

$y = (x^2 - 2x + 1 - 1) + 4$

$y = (x^2 - 2x + 1) + 4 - 1$

$y = (x-1)(x-1) + 3$

$y = (x-1)^2 + 3$

∴ The minimum occurs @ (1, 3)

Example 3

A stone is thrown upwards off a bridge. The height of the stone, H(t) in meters, above the water is given by the equation:

$$H(t) = -5t^2 + 10t + 40$$

- where t is the elapsed time in seconds after the stone was thrown.

a) What was the maximum height attained by the stone and how long did it take to reach this height?

$H(t) = -5t^2 + 10t + 40$

$= -5(t^2 - 2t + 1(-1)) + 40$ (×(-5))

$\left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2 = 1$

$= -5(t^2 - 2t + 1) + 40 + 5$

$\frac{b}{2} = \frac{-2}{2} = -1$

$= -5(t-1)(t-1) + 45$

$= -5(t-1)^2 + 45$

Vertex → (1, 45)
t H

∴ The stone reaches a maximum height of 45m after 1 second.

b) What was the height of the bridge?

Set $t = 0$

$H(0) = -5(0)^2 + 10(0) + 40$

$H(0) = 40m$

∴ The bridge is 40m in height.