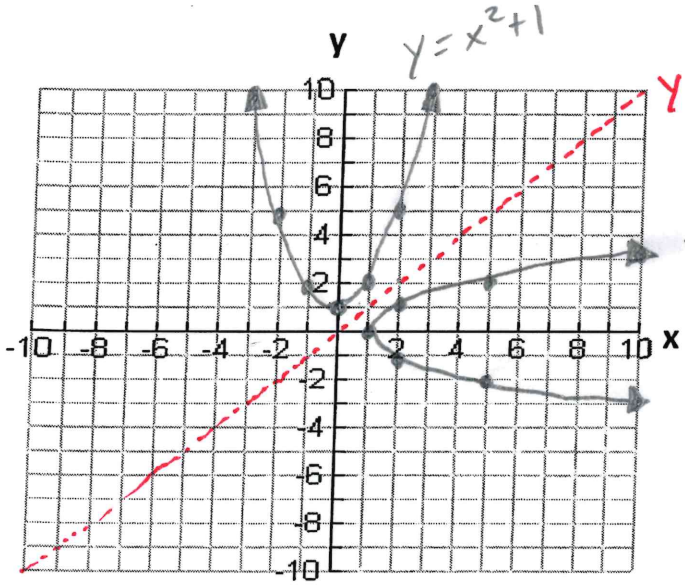
Homework: pg 160 # 1, 4, 6, 10, 11, 12, 13ac

The Inverse of a Quadratic Function

Activity

Graph the function $y = x^2 + 1$ and its inverse on the grid below.

Vertex:
(0, 1)
Step pattern:
1, 3, 5



From the above graphs, we see that the inverse of $y = x^2 + 1$ can be determined by reflecting the entire graph about the $y = x$ line.

We also note that the inverse of $y = x^2 + 1$ is not a function since it would fail the vertical line test.

Example 1

For each function below, determine the domain and range. Then, determine the inverse of the function state its domain and range. Compare.

a) $y = (x + 2)^2 - 5$

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y \geq -5\}$

Inverse

$x = (y+2)^2 - 5$

$(y+2)^2 - 5 = x$

$\sqrt{(y+2)^2} = \pm\sqrt{x+5}$

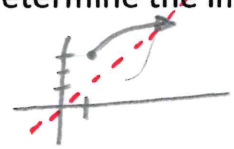
$y + 2 = \pm\sqrt{x+5}$ ← Inverse

$y = \pm\sqrt{x+5} - 2$

Domain: $\{x \in \mathbb{R} \mid x \geq -5\}$

Range: $\{y \in \mathbb{R}\}$

* b) $y = \sqrt{x-1} + 3$



Domain: $\{x \in \mathbb{R} \mid x \geq 1\}$

Range: $\{y \in \mathbb{R} \mid y \geq 3\}$

Inverse

$\sqrt{y-1} + 3 = x$

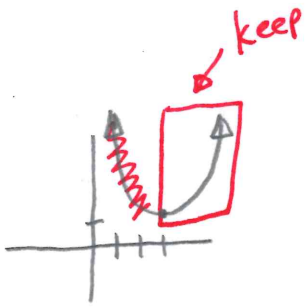
$(\sqrt{y-1})^2 = (x-3)^2$

$y - 1 = (x-3)^2$

$y = (x-3)^2 + 1, x \geq 3$ ← Inverse

Domain: $\{x \in \mathbb{R} \mid x \geq 3\}$

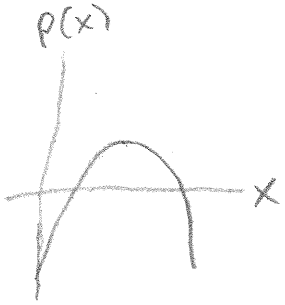
Range: $\{y \in \mathbb{R} \mid y \geq 1\}$



Example 2

The profit of a company can be modeled using the following equation:

$$P(x) = -0.001(x - 150{,}000)^2 + 10{,}000{,}000$$



where

- P(x) is the total profit (in $)
- x is the amount of money invested in advertising (in $)

a) Determine the inverse for this function. ie; isolate the equation for the variable x.

Note: that when finding the inverse of an equation that represents a real world application, we do not swap variables.

$$-0.001(x-150000)^2 + 10000000 = P$$

$$\frac{-0.001(x-150000)^2}{-0.001} = \frac{P-10000000}{-0.001}$$

$$P - 10000000 = -10000000 + P = -(10000000 - P)$$

$$\sqrt{(x-150000)^2} = \pm\sqrt{\frac{-(10000000-P)}{0.001}}$$

$$x - 150000 = \pm\sqrt{\frac{10000000-P}{0.001}}$$

$$x = \pm\sqrt{\frac{10000000-P}{0.001}} + 150000$$

b) How much should be spent on advertising to earn $5,000,000 in profit?

Set $P = 5{,}000{,}000$

$$x = \pm\sqrt{\frac{10000000-5000000}{0.001}} + 150000$$

$$x = \pm 70710.68 + 150000$$

$x = \$220710.68$ or $x = \$79289.32$