The Inverse of a Function

The inverse of a function can be determined from a table of values, from an equation algebraically, or using a graph.

Table of Values

The inverse of a function can be determined from a table of values by swapping x and y variables.

Ex:

Determine the inverse of each relationship represented below.

a)

x	y
1	8
3	5
5	2
7	-1

inverse -------->

x	y
8	1
5	3
2	5
-1	7

b)

x	y
0	18
1	22
2	26
3	30

inverse -------->

x	y
18	0
22	1
26	2
30	3

Equations

The inverse of a function can be determined by swapping x and y variables then isolating for y.

Ex:

Determine the inverse of each relationship below.

a) $f(x) = 2x + 5$

$y = 2x + 5$

$x = 2y + 5$

$2y + 5 = x$

$\frac{2y}{2} = \frac{x}{2} - \frac{5}{2}$

$y = \frac{1}{2}x - \frac{5}{2}$

$f^{-1}(x) = \frac{1}{2}x - \frac{5}{2}$

b) $y = \frac{2}{x-5}$

$\frac{x}{1} = \frac{2}{y-5}$

$\frac{x(y-5)}{x} = \frac{2}{x}$

$y - 5 = \frac{2}{x}$

$y = \frac{2}{x} + 5$

Inverse

c) $f(x) = x^2 + 3$

$y = x^2 + 3$

$y^2 + 3 = x$

$\sqrt{y^2} = \pm\sqrt{x-3}$

$y = \pm\sqrt{x-3}$

Inverse

Note: The symbol $f^{-1}(x)$ represents the inverse of the function $f(x)$ when its' inverse is also a function. Therefore, the symbol $f^{-1}(x)$ cannot be used in question c).

Inverses Graphically

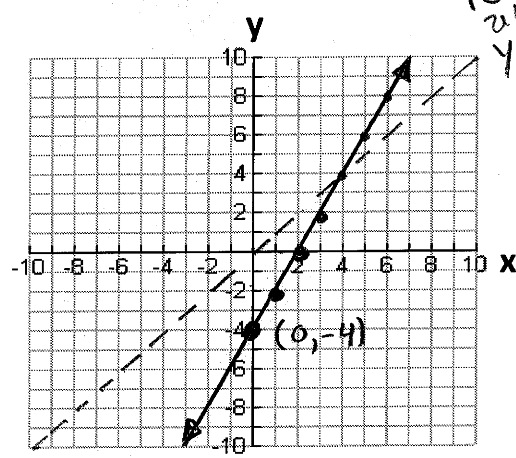
Graph each function below on the left and its inverse on the right.
State the domain and range for each.

reflect about $y=x$

a) $y = 2x - 4$

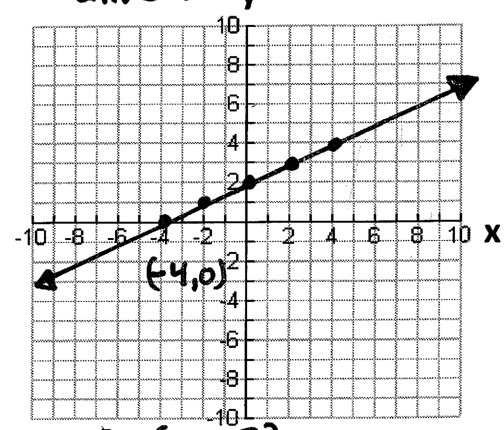
$m = \frac{2}{1}$ $\frac{\text{rise}}{\text{run}}$

$b = -4$



D: $\{x \in \mathbb{R}\}$
R: $\{y \in \mathbb{R}\}$

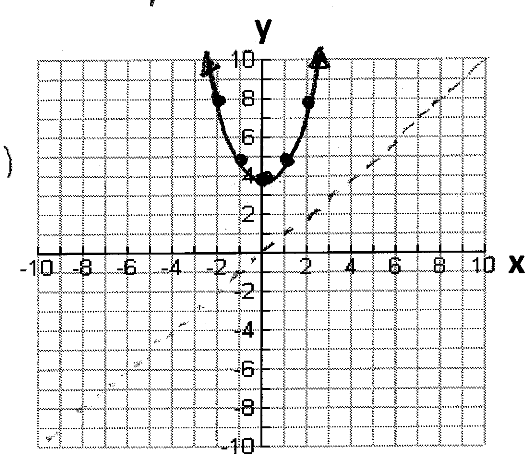
Inverse



D: $\{x \in \mathbb{R}\}$
R: $\{y \in \mathbb{R}\}$

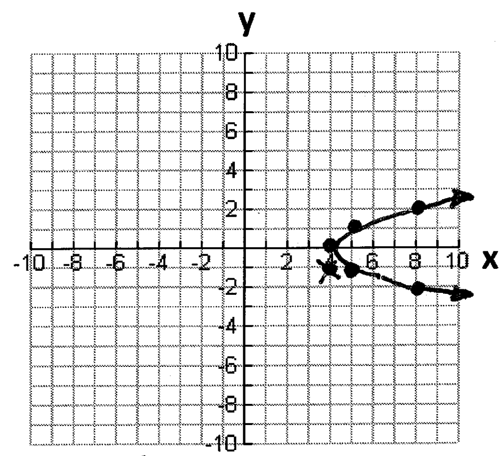
b) $y = x^2 + 4$

$y = (x-0)^2 + 4$
vertex $\to (0,4)$
step pattern: 1, 3, 5



D: $\{x \in \mathbb{R}\}$
R: $\{y \in \mathbb{R} \mid y \geq 4\}$

Inverse



D: $\{x \in \mathbb{R} \mid x \geq 4\}$
R: $\{y \in \mathbb{R}\}$

Some Properties of Inverses (as shown from the graphs above):

1. The inverse is obtained by reflecting the graph about the y = x line; or by interchanging the 'x' and 'y' coordinates of each point.
2. The inverse of a function is not always a function; the inverse in a) is a function while the inverse in b) is not.
3. The domain of the inverse can be determined by examining the range of the original function.
4. The range of the inverse can be determined by examining the domain of the original function.

Homework: pg 46 # 1, 3, 4, 5, 6, 10ace, 16, 17