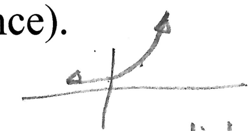
Homework: pg 459 #1ac, 3ace, 4, 5, 6ace, 7, 9, 11, (15)

Geometric Series

The Homework Proposal

"Suppose" we make an offer to pay you to complete your homework. On the first day, we'll pay you 5¢. On the second day, we'll pay you 10¢. On the third day, we'll pay you 20¢, then 40¢, etc… (following a geometric sequence).



Exponential Growth

Given this proposition…

a) How much will we pay you on the 20th day?

$t_n = ar^{n-1}$

$t_n = 5(2)^{n-1}$

set $n = 20$

$t_{20} = 5(2)^{19}$

$t_{20} = 2621440$

You would get \$26214.40

b) How much would we pay you *total* after 20 days?

① $S = 5 + 10 + 20 + 40 + \ldots + 2621440$

multiply by $r = 2$ → ② $2S = 10 + 20 + 40 + \ldots + 2621440 + 5242880$

② - ① $S = -5 + 5242880$

$S = 5242875$

You would get a total of \$52428.75

Create a formula that can be used to determine the sum of the first n terms in a geometric series.

$ar^{n-1} \cdot r$ ①

$S = t_1 + t_2 + t_3 + t_4 + \ldots + t_n$

① $S = a + ar^1 + ar^2 + ar^3 + \ldots + ar^{n-1}$

② $rS = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n$

② - ① $rS - S = -a + ar^n$

$S(r-1) = ar^n - a$

$\frac{S\cancel{(r-1)}}{\cancel{r-1}} = \frac{a(r^n - 1)}{\cancel{r-1}}$

$$\boxed{S_n = \frac{a(r^n - 1)}{r-1}}$$

Sum of a Geometric Series

Example 1

Use the previously derived formula to determine the sum of the following series:

$2 + 8 + 32 + \ldots + 524288$

$t_n = ar^{n-1}$

$t_n = (2)(4)^{n-1}$

$\frac{524288}{2} = \frac{2(4)^{n-1}}{2}$

$262144 = 4^{n-1}$

$4^9 = 4^{n-1}$

$9 = n-1$

$n = 10$

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_{10} = \frac{2(4^{10} - 1)}{4-1}$

$= \frac{2(1048575)}{3}$

$= 699050$

Create a second formula to determine the sum of a geometric series.

$S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{ar^n - a}{r-1}$

$\boxed{S_n = \frac{t_{n+1} - t_1}{r-1}}$

* ar^n is the term after the last term

Example 2

Use the new formula to again determine the sum of the series:

$2 + 8 + 32 + \ldots + 524288$

t_n

$t_{n+1} = 4(524288)$

$= 2097152$

$S_n = \frac{t_{n+1} - t_1}{r-1}$

$= \frac{2097152 - 2}{4-1}$

$= 699050$