Homework: pg 430 # 1, 2ac, 3, 4, 5, 6ace, 7, 8, 9, 11, 20, 22

Introduction to Geometric Sequences

A ***geometric sequence*** is an ordered list of numbers such that each term is produced by multiplying the previous term by a constant.

The following are examples of geometric sequences:

a) 2, 6, 18, 54, …

b) 96, 48, 24, 12, 6, …

c) –3, 6, -12, 24, -48, …

The constant multiplier used to create subsequent terms is referred to as the ' common ratio '; the letter 'r' is used to represent this quantity. We will continue to use the letter 'a' to represent the first term.

Example

For each geometric sequence described above, determine the first term 'a' and the common ratio 'r'.

a) $a = 2$ $\quad r = \frac{t_4}{t_3} = \frac{54}{18}$ $\quad r = 3$

b) $a = 96$ $\quad r = \frac{t_3}{t_2} = \frac{24}{48}$ $\quad r = \frac{1}{2}$

c) $a = -3$ $\quad r = \frac{t_2}{t_1}$ $\quad r = \frac{6}{-3}$ $\quad r = -2$

Consider the following geometric sequence:

5, 10, 20, 40, 80,….

a = 5

r = 2

This sequence can be written out as:

5, 5 x 2, 5 x 2 x 2, 5 x 2 x 2 x 2, 5 x 2 x 2 x 2 x 2, …

or

5×2^0, 5×2^1, 5×2^2, 5×2^3, 5×2^4, …

If we use the variables 'a' and 'r', we get…

ar^0, ar^1, ar^2, ar^3, ar^4, ar^5, ar^6, …

Observing this sequence, it is possible to create a general formula and recursive formula for the nth term of a geometric sequence as follows.

The general term of an Geometric Sequence

$\boxed{t_n = ar^{n-1}}$

where

- **a is the first term**
- **r is the common ratio. It is found by dividing any two consecutive terms in the sequence ie;** $r = \dfrac{t_n}{t_{n-1}}$.
- **n is the term number**

The recursive formula for a Geometric Sequence

$t_1 = a$, $t_n = r \cdot t_{n-1}$ **, where n > 1**

Example 1

Determine the recursive formula, the general term and the 8th term for each geometric sequence:

a) 5, -15, 45,…

$t_1 = 5,\ t_n = -3t_{n-1},\ n > 1$

$t_n = ar^{n-1}$

$t_n = (5)(-3)^{n-1}$

set $n = 8$

$t_8 = 5(-3)^7$

$t_8 = -10935$

b) 1536, 768, 384, ….

$r = \frac{t_3}{t_2} = \frac{384}{768} = \frac{1}{2}$

$t_1 = 1536,\ t_n = \frac{1}{2}t_{n-1},\ n > 1$

$t_n = ar^{n-1}$

$t_n = 1536\left(\frac{1}{2}\right)^{n-1}$

set $n = 8$

$t_8 = 1536\left(\frac{1}{2}\right)^7$

$t_8 = 12$

Example 2

The first term in a geometric sequence is 3 and the 5th term is 48. What is the 10th term?

$t_n = ar^{n-1}$

$t_n = 3r^{n-1}$

set $n = 5$

$t_5 = 3r^4$

$\frac{48}{3} = \frac{3r^4}{3}$

$\pm\sqrt[4]{16} = \sqrt[4]{r^4}$

$\pm 2 = r$

Case 1 ($r = -2$)

$t_n = 3(-2)^{n-1}$

set $n = 10$

$t_{10} = 3(-2)^9$

$t_{10} = -1536$

Case 2 ($r = 2$)

$t_n = 3(2)^{n-1}$

set $n = 10$

$t_{10} = 3(2)^9$

$t_{10} = 1536$

$\therefore$ The tenth term is ± 1536

* Showing "r" can be positive or negative

3, -6, 12, -24, 48

or 3, 6, 12, 24, 48

Example 3

The ninth term is 1792. The 12th term is 14336. What is the 15th term?

$t_n = ar^{n-1}$

$t_9 = ar^8$

① $1792 = ar^8$

$t_{12} = ar^{11}$

② $14336 = ar^{11}$

Isolate a in ①

$\frac{1792}{r^8} = \frac{a\cancel{r^8}}{\cancel{r^8}}$

③ $a = \boxed{\frac{1792}{r^8}}$

sub ③ into ②

$14336 = \frac{1792}{r^8} \cdot \frac{r^{11}}{1}$

$14336 = \frac{1792r^{11}}{r^8}$

$\frac{14336}{1792} = \frac{\cancel{1792}r^3}{\cancel{1792}}$

$\sqrt[3]{r^3} = \sqrt[3]{8}$

④ $r = 2$

sub ④ into ③

$a = \frac{1792}{(2)^8}$

$a = 7$

$t_n = ar^{n-1}$

$t_n = 7(2)^{n-1}$

$t_{15} = 7(2)^{14}$

$t_{15} = 114688$