

Future Value Annuities

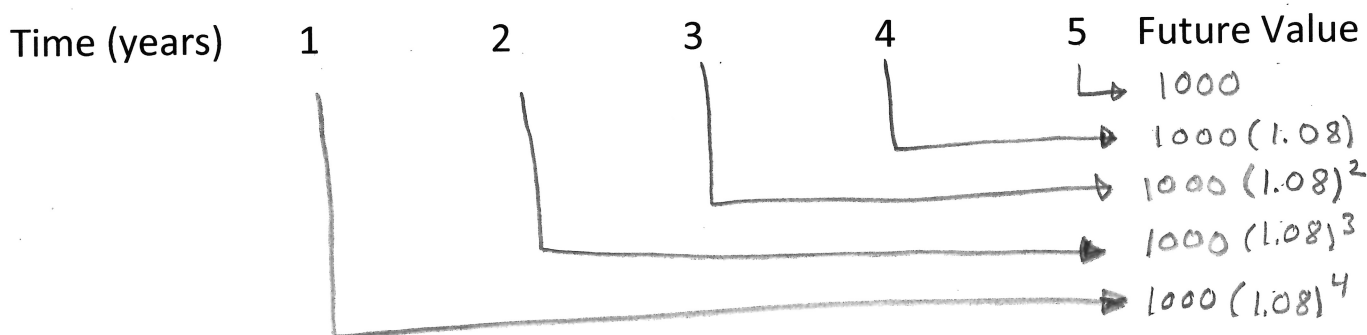
Suppose that instead of making a one-time deposit into a savings account, we make several regular deposits at scheduled intervals into an account that grows as interest accumulates. Pensions often follow this model; this is referred to as a future value annuity.

Example 1



Consider the scenario where we invest \$1000 at the end of each year for 5 years. The account earns 8% interest compounded annually.

Note: When the payments are made at the end of each period, it is referred to as an ordinary annuity (type of future value annuity).



$$A = 1000 + 1000(1.08) + 1000(1.08)^2 + 1000(1.08)^3 + 1000(1.08)^4$$

(future value)

geometric series

$$a = 1000$$

$$r = 1.08$$

$$n = 5$$

$$A = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1000(1.08^5 - 1)}{1.08 - 1}$$

$$= \$5866.60$$

Use the solution from the previous equation to create an equation for the value of a future value annuity where:

- 'A' is the future value of the account.
- 'R' are the regular payments.
- 'i' is the interest rate earned at each period.
- 'n' is the number of deposits or the number of interest payment collections.

$$A = \frac{R[(1+i)^n - 1]}{i}$$

Example 2

Kylo Ren would like to retire in 25 years. He sets up a plan such that he makes \$500 payments each month into an account that earns 6%/a compounded monthly. How much will this account be worth in 25 years when he retires?

$$A = ?$$

$$R = 500$$

$$i = 0.06 \div 12 = 0.005$$

$$n = 25 \times 12 = 300$$

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$= \frac{500[(1.005)^{300} - 1]}{0.005}$$

$$= \$346,496.98$$

Modify the future value annuity equation to isolate it for 'R'.

$$\frac{A}{1} = \frac{R[(1+i)^n - 1]}{i}$$

$$\frac{R[(1+i)^n - 1]}{(1+i)^n - 1} = \frac{Ai}{(1+i)^n - 1}$$

$$R = \frac{Ai}{(1+i)^n - 1}$$

Example 3

Princess Leia would like to retire in 30 years with a lump sum of 2 million dollars. How much money will she need to deposit into her pension plan each month if she thinks that she can earn 8%/a from the Rebel Alliance Investors?

$$A = 2000000$$

$$R = ?$$

$$i = 0.08 \div 12 = 0.00\bar{6}$$

$$n = 30 \times 12 = 360$$

$$R = \frac{Ai}{(1+i)^n - 1}$$

$$= \frac{2000000(0.00\bar{6})}{(1.00\bar{6})^{360} - 1}$$

$$= \$1341.96/\text{month}$$