Practice Worksheet +
pg 511 #1c, 2abc, 3, 5cd, 6a, 7ad, 8, 9, (10), (14)

Homework: ~~pg 511 # 1c, 2ac, 3, 5d, 6, 7, 9, (10), (14)~~

Future Value Annuities

Suppose that instead of making a one-time deposit into a savings account, we make several regular deposits at scheduled intervals into an account that grows as interest accumulates. Pensions often follow this model; this is referred to as a future value annuity.

Example 1

deposits interest (compound)

Consider the scenario where we invest $1000 at the end of each year for 5 years. The account earns 8% interest compounded annually.

Note: When the payments are made at the end of each period, it is referred to as an ordinary annuity (type of future value annuity).

Time (years)	1	2	3	4	5	Future Value
						1000
						$1000(1.08)$
						$1000(1.08)^2$
						$1000(1.08)^3$
						$1000(1.08)^4$

A (future value) $= \underbrace{1000 + 1000(1.08) + 1000(1.08)^2 + 1000(1.08)^3 + 1000(1.08)^4}_{\text{geometric series}}$

$a = 1000$

$r = 1.08$

$n = 5$

$$A = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1000(1.08^5 - 1)}{1.08 - 1}$$

$= \$5866.60$

Use the solution from the previous equation to create an equation for the value of a future value annuity where:

- 'A' is the future value of the account.
- 'R' are the regular payments.
- 'i' is the interest rate earned at each period.
- 'n' is the number of deposits or the number of interest payment collections.

$$A = \frac{R[(1+i)^n - 1]}{i}$$

Example 2

Kylo Ren would like to retire in 25 years. He sets up a plan such that he makes $500 payments each month into an account that earns 6%/a compounded monthly. How much will this account be worth in 25 years when he retires?

$A = ?$

$R = 500$

$i = 0.06 \div 12 = 0.005$

$n = 25 \times 12 = 300$

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$= \frac{500[(1.005)^{300} - 1]}{0.005}$$

$$\doteq 346496.98$$

Modify the future value annuity equation to isolate it for 'R'.

$$\frac{A}{1} = \frac{R[(1+i)^n - 1]}{i}$$

$$\frac{R[\cancel{(1+i)^n - 1}]}{\cancel{(1+i)^n - 1}} = \frac{Ai}{(1+i)^n - 1}$$

$$\boxed{R = \frac{Ai}{(1+i)^n - 1}}$$

Example 3

Princess Leia would like to retire in 30 years with a lump sum of 2 million dollars. How much money will she need to deposit into her pension plan each month if she thinks that she can earn 8%/a from the Rebel Alliance Investors?

$A = 2000000$

$R = ?$

$i = 0.08 \div 12 = 0.00\bar{6}$

$n = 30 \times 12 = 360$

$$R = \frac{Ai}{(1+i)^n - 1}$$

$$= \frac{2000000(0.00\bar{6})}{(1.00\bar{6})^{360} - 1}$$

$$= \$1341.96/\text{month}$$