Practice

~~Review~~: Future Value Annuities

Future Value Annuity – a series of regular payments are made at regular intervals into an investment that grows with the addition of collected interest. For simple annuities, the compounding interest period coincides with the regular payment schedule. The future value of this investment is related to the payments and interest rate according to the following equation(s):

$$A = \frac{R[(1+i)^n - 1]}{i} \qquad \text{and} \qquad R = \frac{Ai}{(1+i)^n - 1}$$

where

- A is the future value of the investment
- R is the regular payments amount ($)
- i is the interest rate (expressed as a decimal)
- n is the number of payments or compounded interest collections

These equations are commonly used to analyze the process of building a **pension** for **retirement**.

Example 1

Shannon makes monthly payments of $400/month over a 25 year career. The interest rate is 3%/a compounded monthly. How much will the investment be worth when she retires?

$A = ?$

$R = \$400$

$i = 0.03 \div 12 = 0.0025$

$n = 25 \times 12 = 300$

$A = \dfrac{R[(1+i)^n - 1]}{i}$

$= \dfrac{400[1.0025^{300} - 1]}{0.0025}$

$= \$178403.13$

Example 2

Marley is an ambitious investor. He wants to be a millionaire when he retires in 25 years. What will his monthly payments need to be if he thinks that he can earn 6%/a compounded monthly with some savvy investing?

$A = 1000000$

$R = ?$

$i = 0.06 \div 12 = 0.005$

$n = 25 \times 12 = 300$

$$R = \frac{Ai}{(1+i)^n - 1}$$

$$= \frac{(1000000)(0.005)}{[(1.005)^{300} - 1]}$$

$$= \$1443.01/\text{month}$$

Example 3 (refer to Ex. 1)

Shannon later decides that she would also like to be a millionaire when she retires. How much longer will she need to work if her payments and interest rate of return on the investments do not change?

$A = 1000000$

$R = 400$

$i = 0.0025$

$n = ?$

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$\frac{1000000}{1} = \frac{400(1.0025^n - 1)}{0.0025}$$

$$\frac{\cancel{400}(1.0025^n - 1)}{\cancel{400}} = \frac{1000000(0.0025)}{400}$$

$$1.0025^n = \frac{1000000(0.0025)}{400} + 1$$

$$1.0025^n = 7.25$$

$$\log 1.0025^n = \log 7.25$$

$$\frac{n \cancel{\log 1.0025}}{\cancel{\log 1.0025}} = \frac{\log 7.25}{\log 1.0025}$$

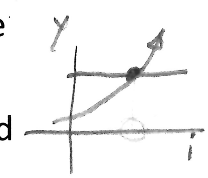
$$n \doteq 793 \text{ months}$$

$$n \doteq 66.12 \text{ years}$$

$\therefore$ She'd need to work an extra 41 years.

Example 4

Shannon realizes that if she wants to become a millionaire at retirement that she will need to shop around to find a better interest rate. If she maintains her monthly payments of \$400 over a 25 year career, what interest rate will she need to obtain to reach her goal? Why is this such a difficult question to answer?



$A = 1000000$

$R = 400$

$i = ?$

$n = 25 \times 12 = 300$

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$1000000 = \frac{400[(1+i)^{300} - 1]}{i}$$

Use technology!

$$y = \frac{400((1+i)^{300} - 1)}{i}$$

$$y = 1000000$$

$$i = 0.011323$$

$\therefore$ The annual interest rate is 12×0.011323

$= 13.6\%/a$