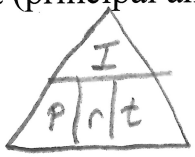
Review Worksheet – Financial Mathematics

1. Owen puts $1200 into an investment that earns 6%/a simple interest. How long will it take until the investment (principal and interest) is worth $1920?

$A = P + I$

$1920 = 1200 + I$

$I = 720$



$t = \frac{I}{Pr}$

$= \frac{720}{1200(0.06)}$

$= 10$ years

2. Mr. and Mrs. Hoover put a one-time deposit of $2500 into an investment to be used 16 years later by their daughter for her post-secondary education. If the investment is expected to earn 8%/a compounded quarterly, how much will the investment be worth at the end of its term?

$A = ?$

$P = 2500$

$i = 0.08 \div 4 = 0.02$

$n = 16 \times 4 = 64$

$A = P(1+i)^n$

$= 2500(1.02)^{64}$

$= \$8878.73$

3. Lionel anticipates that he'll need to replace the roof on his house at a cost of $12000 in 10 years. How much money should he put into an investment today to support this replacement cost in the future if he expects to earn 5%/a interest compounded semi-annually?

$A = 12000$

$P = ?$

$i = 0.05 \div 2 = 0.025$

$n = 10 \times 2 = 20$

$P = \frac{A}{(1+i)^n}$

$= \frac{12000}{(1.025)^{20}}$

$= \$7323.25$

4. Blake has just been hired by an engineering firm. For his pension plan, Blake will be contributing $400 of his monthly salary into a fund while his employer matches this amount. If Blake hopes to retire in 25 years and expects to earn 6%/a interest compounded monthly, what will be the value of his pension when he retires?

$A = ?$

$R = 800$

$i = 0.06 \div 12 = 0.005$

$n = 25 \times 12 = 300$

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$= \frac{800[(1.005)^{300} - 1]}{0.005}$$

$= \$554395.17$

5. Mrs. Flemington has just retired after working at a car manufacturer for 30 years. During this time, she managed to save up $750000 for her pension. If Mrs. Flemington plans to withdraw an equal amount of money from her savings each month for the next 25 years, how much will she be able to withdraw? Assume that the interest rate is 9%/a compounded monthly.

$P = 750000$

$R = ?$

$i = 0.09 \div 12 = 0.0075$

$n = 25 \times 12 = 300$

$$R = \frac{Pi}{1 - (1+i)^{-n}}$$

$$= \frac{750000(0.0075)}{[1 - (1.0075)^{-300}]}$$

$= \$6293.97/\text{month}$

6. Mackenzie expects that she will need about $2000 a month for 20 years after she retires. If the interest rate is expected to be about 4%/a compounded monthly while she is retired, how much money will she need to have saved away to meet her objectives?

$P = ?$

$R = 2000$

$i = 0.04 \div 12 = 0.00\overline{3}$

$n = 20 \times 12 = 240$

$$P = \frac{R[1 - (1+i)^{-n}]}{i}$$

$$= \frac{2000[1 - (1.00\overline{3})^{-240}]}{0.00\overline{3}}$$

$= \$330043.72$

7. Saul is planning to purchase a house that costs $740000. He has $40000 to be used for a downpayment. Assume that Saul has shopped around and found a mortgage with an interest rate of 2.7%/a.

a) Determine the equivalent monthly interest rate.

semi-annual rate $= 0.027 \div 2 = 0.0135$

$\sqrt[6]{(1+i)^6} = \sqrt[6]{1.0135}$

$i = 0.002237447$

b) What will the monthly payments be if Saul opts for a mortgage with an amortization period of 25 years?

$P = 700000$

$R = ?$

$i = 0.002237447$

$n = 25 \times 12 = 300$

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

$$= \frac{700000(0.002237447)}{1-(1.002237447)^{-300}}$$

$= \$3205.91/\text{month}$

c) How much of the principal will remain after 5 years of payments?

$P = ?$

$R = 3205.91$

$i = 0.002237447$

$n = 20 \times 12 = 240$

$$P = \frac{R[1-(1+i)^{-n}]}{i}$$

(time remaining)

$$= \frac{3205.91[1-(1.002237447)^{-240}]}{0.002237447}$$

$= \$594834.30$

d) How much was paid in interest over the entire 25 year amortization period?

Total payments $= 300 \times 3205.91$

$= 961773$

Interest $= 961773 - 700000$

$= \$261773$

e) If Saul increase his payments by $300, how much quicker could he pay off the mortgage? Under this plan, how much money will he save in interest?

$P = 700000$

$R = 3505.91$

$i = 0.002237447$

$n = ?$

$$P = \frac{R[1-(1+i)^{-n}]}{i}$$

$$\frac{700000}{1} = \frac{3505.91[1-(1.002237447)^{-n}]}{0.002237447}$$

see next page!

$$\frac{700000(0.00223744^{7})}{3505.91} = \frac{3505.91\left[1-(1.00223744^{7})^{-n}\right]}{3505.91}$$

$$0.44673505^{6} = 1-(1.00223744^{7})^{-n}$$

$$\frac{-0.55326494^{4}}{-1} = \frac{-(1.00223744^{7})^{-n}}{-1}$$

$$0.55326494^{4} = 1.00223744^{7-n}$$

$$\frac{\log(0.55326494^{4})}{\log(1.00223744^{7})} = \frac{-n\log(1.00223744^{7})}{\log(1.00223744^{7})}$$

$$-n = -265$$

$$n = 265$$

(≈ 22 years 1 month)

2 years and 11 months quicker

total payments $= 265 \times (3505.91)$

$= \$929066$

From part d)

Savings $= 961773 - 929066$

$= \$32707$