

## Quadratic Functions and its Families

Quadratic functions are often expressed in any one of three ways; standard form, vertex form, and factored form.

(x-int Form)

From each form, we can extract unique details about the graph of the corresponding parabola.

$$y = 2(x-3)^2 + 5$$

Standard form	Vertex Form	Factored Form
$y = ax^2 + bx + c$	$y = a(x - h)^2 + k$	$y = a(x - x_1)(x - x_2)$
$a \rightarrow$ direction of opening and step pattern	$a \rightarrow$ direction of opening and step pattern	$a \rightarrow$ direction of opening and step pattern
$c \rightarrow$ y-intercept	$(h, k) \rightarrow$ vertex	$x_1$ and $x_2$ are the x-intercepts

A quadratic function can be changed to the different forms using algebraic techniques;

- Standard Form  $\rightarrow$  expand and simplify
- Vertex Form  $\rightarrow$  completing the square
- Factored Form  $\rightarrow$  factoring

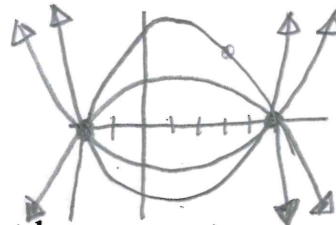
To determine the family of quadratic functions for given criteria means to create a general equation that allows for some variation to account for all possible cases.

### Example 1

$$x - (-2) \\ x + 2$$

Determine the family of quadratic functions that have x-intercepts of -2 and 5.

$$y = a(x - x_1)(x - x_2) \\ y = a(x + 2)(x - 5)$$

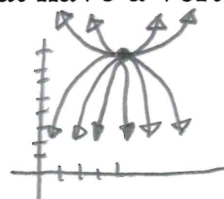


### Example 2

Determine the family of quadratic functions that have a vertex of (4, 8).

x y

$$y = a(x - h)^2 + k \\ y = a(x - 4)^2 + 8$$



### Example 3

Determine the specific quadratic function that has x-intercepts of 1 and 5 and passes through the point (3, 12).

$$y = a(x - x_1)(x - x_2)$$

$$y = a(x - 1)(x - 5)$$

Sub in (3, 12)

$$12 = a(3 - 1)(3 - 5)$$

$$12 = a(2)(-2)$$

$$12 = -4a$$

$$\frac{-4}{-4} = \frac{-4a}{-4}$$

$$a = -3$$

$$y = -3(x - 1)(x - 5)$$

### Example 4

Determine the equation of the parabola that has a vertex at (5, -1) and passes through the point (6, 3).

$$y = a(x - h)^2 + k$$

$$y = a(x - 5)^2 - 1$$

Sub in (6, 3)

$$3 = a(6 - 5)^2 - 1$$

$$3 = a - 1$$

$$3 + 1 = a$$

$$a = 4$$

$$y = 4(x - 5)^2 - 1$$

### Example 5

Express the following quadratic function in all three forms. Fill in the information below and graph the function.

Standard form

$$y = -2x^2 - 12x - 10$$

$$y = -2(x^2 + 6x) - 10$$

$$y = -2(x^2 + 6x + 9 - 9) - 10$$

$$y = -2(x^2 + 6x + 9) - 10 + 18$$

$$y = -2(x + 3)(x + 3) + 8$$

$$y = -2(x + 3)^2 + 8$$

Vertex form

$$y = -2x^2 - 12x - 10$$

$$y = -2(x^2 + 6x + 5)$$

$$y = -2(x + 1)(x + 5)$$

factored form

Direction of opening	down
Step pattern	-2, -6, -10
Maximum/Minimum	Maximum
x-intercept(s)	-1 & -5
y-intercept	-10
Vertex	(-3, 8)

