

Unit 4 - Exponentials Review - Worksheet

$$1. a) (-3xy)^4 \\ = (-3)^4 x^4 y^4 \\ = 81x^4y^4$$

$$b) \frac{6x^3y^6}{3x^2yz^3} \\ = \frac{2xy^5}{z^3}$$

$$c) \left(\frac{x}{3}\right)^{-4} \\ = \left(\frac{3}{x}\right)^4 \\ = \frac{3^4}{x^4} = \frac{81}{x^4}$$

$$d) 2(x^2y)^3 \cdot 3xy^{-3} \\ = 2(x^2)^3(y^3) \cdot 3x \cdot \frac{1}{y^3} \\ = \frac{2x^6y^3}{1} \cdot \frac{3x}{y^3} \\ = \frac{6x^7y^3}{y^3} \\ = 6x^7$$

$$e) 3x^{-4} \\ = 3 \cdot \frac{1}{x^4} \\ = \frac{3}{x^4}$$

$$f) \left(\frac{2x}{3y^2}\right)^{-2} \\ = \left(\frac{3y^2}{2x}\right)^2 \\ = \frac{(3y^2)^2}{(2x)^2} = \frac{9y^4}{4x^2}$$

$$g) \frac{x^2y^{-4}z^{-2}}{p^{-1}q^3} \\ = \frac{x^2p}{y^4z^2q^3}$$

$$h) \frac{1}{2^{-4}} \\ = 2^4 \\ = 16$$

$$i) -2.586^0 \\ = -1$$

$$2. a) 8^{4/3} \\ = (\sqrt[3]{8})^4 \\ = (2)^4 \\ = 16$$

$$b) (64)^{2/3} \\ = (\sqrt[3]{64})^2 \\ = (4)^2 \\ = 16$$

$$c) (-125)^{5/3} \\ = (\sqrt[3]{-125})^5 \\ = (-5)^5 \\ = -3125$$

$$d) (4x^2)^{3/2} \\ = 4^{3/2} (x^2)^{3/2} \\ = (\sqrt[2]{4})^3 x^3 \\ = (2)^3 x^3 \\ = 8x^3$$

$$\begin{aligned}
 3.a) \quad 2^{2x+1} &= 8 \\
 2^{2x+1} &= 2^3 \\
 2x+1 &= 3 \\
 2x &= 3-1 \\
 2x &= 2 \\
 \frac{2x}{2} &= \frac{2}{2} \\
 \boxed{x=1}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad 16^{x+1} &= 2^{x+13} \\
 (2^4)^{x+1} &= 2^{x+13} \\
 2^{4x+4} &= 2^{x+13} \\
 4x+4 &= x+13 \\
 4x-x &= 13-4 \\
 3x &= 9 \\
 \frac{3x}{3} &= \frac{9}{3} \\
 \boxed{x=3}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad 2^{3x} &= \frac{1}{64} \\
 2^{3x} &= 2^{-6} \\
 \frac{3x}{3} &= \frac{-6}{3} \\
 \boxed{x=-2}
 \end{aligned}$$

$$\begin{aligned}
 d) \quad 2^{2x} - 6(2^x) &= 16 \\
 (2^x)^2 - 6(2^x) - 16 &= 0 \\
 \text{Let } n &= 2^x \\
 n^2 - 6n - 16 &= 0 \\
 (n-8)(n+2) &= 0 \\
 n=8 \text{ or } n &= -2
 \end{aligned}$$

Case 1

$$\begin{aligned}
 n &= 8 \\
 2^x &= 8 \\
 \boxed{x=3}
 \end{aligned}$$

Case 2

$$\begin{aligned}
 n &= -2 \\
 2^x &= -2 \\
 \text{No Sol}^n
 \end{aligned}$$

4. Done on graph (back of this solution set).

5.a

1st diff.	x	y	1st diff.	2nd diff.
2 <	3	6	> 16	> 8
2 <	5	22	> 24	> 8
2 <	7	46	> 32	> 8
2 <	9	78	> 40	
	11	118		

i. Quadratic relationship

b)

1st diff.	x	y	1st diff ratios
1 <	0	0	> 2
1 <	1	2	> 3
1 <	2	8	> 3
1 <	3	26	> 3
1 <	4	80	> 54

ii. Exponential relationship

c)

1st diff.	x	y	1st diff.
-2 <	4	15	> 5
-2 <	2	20	> 5
-2 <	0	25	> 5
-2 <	-2	30	> 5
	-4	35	

iii. Linear relationship.

6. a)

Exponential decay

$$y = a(1-r)^x$$

$$V = 48500(1-0.08)^t$$

$$V = 48500(0.92)^t$$

b)

set $t = 10$

$$V = 48500(0.92)^{10}$$

$$V \approx 21067.84$$

7. a)

Half-life

$$y = a(0.5)^{t/h}$$

$$D = 30(0.5)^{t/10}$$

b)

set $t = 25$

$$D = 30(0.5)^{25/10}$$

$$D \approx 5.3 \text{ cm}$$

8. a)

Exponential growth

$$y = a(1+r)^x$$

$$P = 3.49(1+0.023)^t$$

$$P = 3.49(1.023)^t$$

b)

set $t = 20$ (20 years after 2020)

$$P = 3.49(1.023)^{20}$$

$$P \approx 5.50$$

c)

set $t = -75$

$$P = 3.49(1.023)^{-75}$$

$$P \approx 0.63$$

9.

Half-life

$$y = a(0.5)^{t/h}$$

$$y = 320(0.5)^{t/5700}$$

$$\text{set } y = 200$$

$$\frac{200}{320} = \frac{320(0.5)^{t/5700}}{320}$$

$$\frac{5}{8} = 0.5^{t/5700}$$

$$\log\left(\frac{5}{8}\right) = \log\left(0.5^{t/5700}\right)$$

$$\log\left(\frac{5}{8}\right) = \frac{t}{5700} \log(0.5)$$

$$\frac{\log(0.5)}{\log(5/8)} = \frac{t}{5700} \frac{\log(0.5)}{\log(0.5)}$$

$$t \frac{\log(0.5)}{\log(5/8)} = \frac{5700 \log(5/8)}{\log(0.5)}$$

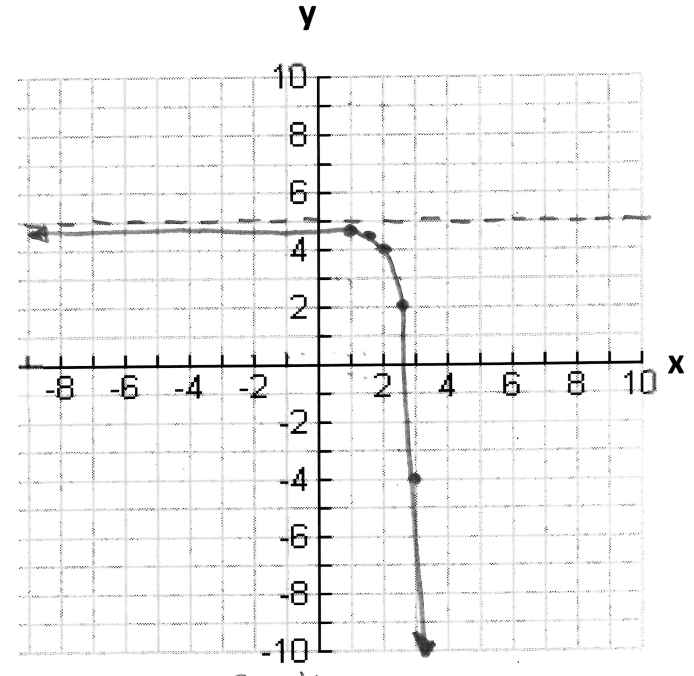
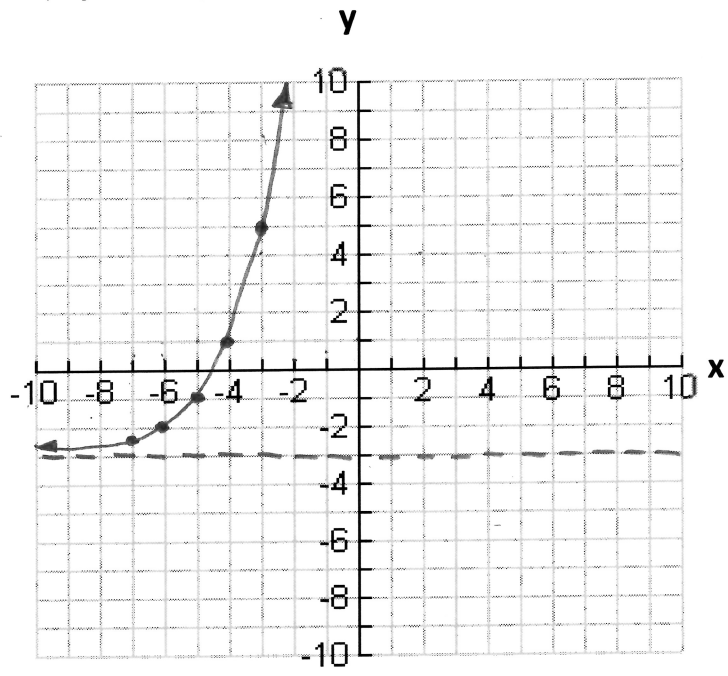
$$t \approx 3865 \text{ years}$$

4. Graph the following exponential functions then state the domain and range.

* Note that $a = -1$

a) $y = 2(2)^{x+5} - 3$

b) $y = -3^{2(x-2)} + 5$



parent function

x	y = 2 ^x
-2	1/4
-1	1/2
0	1
1	2
2	4

h [$k = 1$
 $d = -5$
 $a = 2$
 $c = -3$]

H.A. @ $y = -3$

Domain: $\{x \in \mathbb{R}\}$
 Range: $\{y \in \mathbb{R} \mid y > -3\}$

parent function

x	y = 3 ^x
-2	1/9
-1	1/3
0	1
1	3
2	9

h [$k = 2$
 $d = 2$
 $a = -1$
 $c = 5$]

H.A. @ $y = 5$

Domain $\{x \in \mathbb{R}\}$
 Range $\{y \in \mathbb{R} \mid y < 5\}$