

EXAM REVIEWUNIT 1- Sequences and Series

- Determine the first 4 terms of each of the following sequences for which the general term is given:
  - $t_n = 3n + 1$
  - $t_n = n^2 - 3$
  - $f(n) = 2^n + 1$
  - $t_n = \frac{2n}{n+1}$
  - $t_n = n^3 - 2$
  - $t_n = \frac{n^2 + n}{2}$
- Determine  $t_3$ ,  $t_4$ , and  $t_5$  for the sequence given by  $t_1 = 1$ ,  $t_2 = 3$ ,  $t_k = 2t_{k-1} - 3t_{k-2}$ ,  $k > 2$ .
- Determine a general term for each of the following sequences.
  - 2, 6, 10, 14, ...
  - 2, 6, 18, 54, ...
  - 13, 8, 3, -2, ...
  - 0,  $\frac{1}{4}$ ,  $\frac{2}{9}$ ,  $\frac{3}{16}$ , ...
- Each of the following sequences is either arithmetic or geometric. In each case, determine the type of sequence, the general term, and the indicated term.
  - 5, 8, 11, 14, ... Determine  $t_{20}$ .
  - 3, 9, 27, 81, ... Determine  $t_8$ .
  - 54, 48, 42, 36, ... Determine  $t_{15}$ .
- Each of the following sequences is either arithmetic or geometric. In each case determine the type of sequence and the number of terms.
  - 5, 12, 19, ..., 222
  - 8, 24, 72, ..., 5832
  - 81, 27, 9, ...,  $\frac{1}{243}$
  - 8, 5, 2, ..., -64
- Determine  $t_{20}$  in the arithmetic sequence where  $t_6 = 28$  and  $t_{11} = 63$ .
- A superball is dropped from a height of 60 m. The ball bounces back  $\frac{4}{5}$  of the distance it falls.
  - To what height does the ball bounce after the first bounce?
  - To what height, to one decimal place, does the ball bounce after the

sixth bounce?

8. Each of the following series is either arithmetic or geometric. In each case, determine the type of series and the sum of each series.
- $1 - 2 + 4 - \dots$  Find  $S_{100}$ .
  - $5 + 9 + 13 + \dots + 213$
9. The sum of  $n$  terms of the geometric series  $1 - 3 + 9 - \dots$  is  $-132860$ . What is the value of  $n$ ?
10. How many terms in the arithmetic sequence  $7, 14, 21, \dots$  are between 40 and 28 001?

## UNIT 2- Rational Expressions

Restrictions, Multiplication and Division of Rational Expressions:

Examples: Find all numbers for which the rational expression  $\frac{x+2}{x^2-2x-3}$  is undefined.

Set the denominator equal to 0 and solve.

$$\begin{aligned} x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0 \\ x-3 = 0 \quad \text{or} \quad x+1 &= 0 \\ x = 3 \quad \text{or} \quad x &= -1 \end{aligned}$$

The expression is undefined when the denominator is equal to 0, or when  $x = 3$  or  $x = -1$ .

Multiply and simplify:

$$\begin{aligned} \frac{x^2+4x+4}{x^2-9} \cdot \frac{x-3}{x+2} &= \frac{(x^2+4x+4)(x-3)}{(x^2-9)(x+2)} \\ &= \frac{(x+2)\cancel{(x+2)}\cancel{(x-3)}}{(x+3)\cancel{(x-3)}(x+2)} \\ &= \frac{x+2}{x+3} \end{aligned}$$

Divide and simplify:

$$\begin{aligned} \frac{x+2}{x^2-16} \div \frac{x^2+x-2}{x^2+3x-4} &= \frac{x+2}{x^2-16} \cdot \frac{x^2+3x-4}{x^2+x-2} \\ &= \frac{(x+2)(x^2+3x-4)}{(x^2-16)(x^2+x-2)} \\ &= \frac{\cancel{(x+2)}\cancel{(x+4)}\cancel{(x-1)}}{\cancel{(x+4)}(x-4)(x+2)\cancel{(x-1)}} \\ &= \frac{1}{x-4} \end{aligned}$$

Find all numbers for which the rational expression is undefined.

- |                                    |                                |
|------------------------------------|--------------------------------|
| 1. $\frac{-2}{5x}$ _____           | 2. $\frac{7}{x-3}$ _____       |
| 3. $\frac{4}{3y+2}$ _____          | 4. $\frac{x^2-4}{5x-10}$ _____ |
| 5. $\frac{x^2+6}{x^2-3x-10}$ _____ | 6. $\frac{x-8}{5}$ _____       |

1. Simplify and state the restrictions on the following rational expressions:

a) $\frac{9x+18}{x^2-4x-21}$	b) $\frac{-x^3+5x^2-6x}{x^3-3x^2}$	c) $\frac{x^3-6x^2+8x}{6x^2-24x}$	d) $\frac{2x^2-15x-27}{3x^2-25x-18}$
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e)  $\frac{x^2+2x-3}{x^2-4x+3}$

f)  $\frac{4x-8}{x^2-4}$

g)  $\frac{6x^2-6x}{x^2+2x-3}$

Multiply and simplify.

7.  $\frac{8x^3}{5x} \cdot \frac{10}{x}$  \_\_\_\_\_

8.  $\frac{5x^2y}{6} \cdot \frac{2}{xy^3}$  \_\_\_\_\_

9.  $\frac{t^2}{t^2-3t} \cdot \frac{t^2-7t+12}{t^2-16}$  \_\_\_\_\_

10.  $\frac{a^2-25}{a^2} \cdot \frac{a^2-2a}{a^2+3a-10}$  \_\_\_\_\_

11.  $\frac{x^2+8x+15}{x^2-1} \cdot \frac{x+1}{x+5}$  \_\_\_\_\_

12.  $\frac{10a^2}{4a^2-a-3} \cdot \frac{4a-4}{2a}$  \_\_\_\_\_

13.  $\frac{6b+6}{b-3} \cdot \frac{b^2-8b+15}{b^2-b-2}$  \_\_\_\_\_

14.  $\frac{x^4-81}{x^4-16} \cdot \frac{x^2+4}{x^2-9}$  \_\_\_\_\_

Divide and simplify.

15.  $\frac{5}{8} \div \frac{3}{4}$  \_\_\_\_\_

16.  $\frac{t}{4} \div \frac{t}{12}$  \_\_\_\_\_

17.  $\frac{a+5}{a-1} \div \frac{6a+30}{a}$  \_\_\_\_\_

18.  $\frac{x^2-49}{x} \div \frac{x+7}{x-2}$  \_\_\_\_\_

19.  $\frac{x^2-64}{2x+16} \div \frac{x-8}{5}$  \_\_\_\_\_

20.  $\frac{a+b}{3a} \div \frac{a^2-b^2}{9a^3}$  \_\_\_\_\_

21.  $\frac{c^2+4c}{c^2-c-20} \div \frac{c}{c-5}$  \_\_\_\_\_

22.  $\frac{3y^2+y-2}{3y^2-8y+4} \div \frac{y^1-y-56}{y^3+5y-14}$  \_\_\_\_\_

23.  $\frac{x^2+10x+21}{x^2+5x+4} \div \frac{x^3+7x^2}{x^2+4x}$  \_\_\_\_\_

24.  $\frac{5t^2-50t-40}{10t-40} \div \frac{t^2-5t-14}{t^2-8t+7}$  \_\_\_\_\_

Addition and Subtraction of Rational Expressions:

Example: Do this calculation.

$$\begin{aligned} & \frac{5x}{x^2 - 3x - 4} - \frac{2x}{x^2 - 6x + 8} \\ &= \frac{5x}{(x-4)(x+1)} - \frac{2x}{(x-4)(x-2)}, \text{ LCM} = (x-4)(x-2)(x+1) \\ &= \frac{5x}{(x-4)(x+1)} \cdot \frac{x-2}{x-2} - \frac{2x}{(x-4)(x-2)} \cdot \frac{x+1}{x+1} \\ &= \frac{5x(x-2) - 2x(x+1)}{(x-4)(x-2)(x+1)} \\ &= \frac{5x^2 - 10x - 2x^2 - 2x}{(x-4)(x-2)(x+1)} \\ &= \frac{3x^2 - 12x}{(x-4)(x-2)(x+1)} \\ &= \frac{3x(x-4)}{(x-4)(x-2)(x+1)} \\ &= \frac{3x}{(x-2)(x+1)} \end{aligned}$$

Add or subtract. Simplify.

1.  $\frac{x-1}{x+3} + \frac{x+7}{x+3}$  \_\_\_\_\_

2.  $\frac{x-1}{x+6} + \frac{x+3}{x-2}$  \_\_\_\_\_

3.  $\frac{a^2}{a-4} + \frac{16}{4-a}$  \_\_\_\_\_

4.  $\frac{4y}{y^2 - y - 2} - \frac{5y}{y^2 + y - 6}$  \_\_\_\_\_

5.  $\frac{3x+2}{x-1} - \frac{x+5}{x-1}$  \_\_\_\_\_

6.  $\frac{4}{a+2} + \frac{a+1}{a^2-4} - \frac{3}{a-2}$  \_\_\_\_\_

7.  $\frac{y-5}{3y+9} - \frac{y+1}{2y+6}$  \_\_\_\_\_

8.  $\frac{5}{a} + \frac{3}{-a}$  \_\_\_\_\_

9.  $\frac{x+1}{x^2-7x+10} + \frac{3}{x^2-x-2}$  \_\_\_\_\_

10.  $\frac{b-3}{b^2-9} + \frac{b+3}{b^2+6b+9}$  \_\_\_\_\_

### UNIT 3 – Quadratic Functions

1. Complete the table with the equivalent functions.

$y = ax^2 + bx + c$	$y = a(x-r)(x-s)$	$y = a(x-h)^2 + k$
	$y = (x-2)(x-1)$	

$y = 2x^2 + 8x - 24$		
		$y = (x - 3)^2$

2. Determine the equation of the function that has x-intercepts -2 and 5 and goes through the point ( 4, 6).

3. State the number of roots for the following quadratics.

$D = b^2 - 4ac$  Show your work. (Recall: , D→ Discriminant)

$x^2 + 2x + 2 = 0$  a) \_\_\_\_\_

$x^2 - 4x + 4 = 0$  b) \_\_\_\_\_

$x^2 + x - 2 = 0$  c) \_\_\_\_\_

4. Complete the square.

a)  $y = x^2 - 6x + 10$

b)  $y = 2x^2 - 16x + 11$

c)  $y = -3x^2 + 3x - 5$

5. Solve each quadratic equation using any method:

a)  $x^2 - 5 = 0$

b)  $x^2 - 7x + 12 = 0$

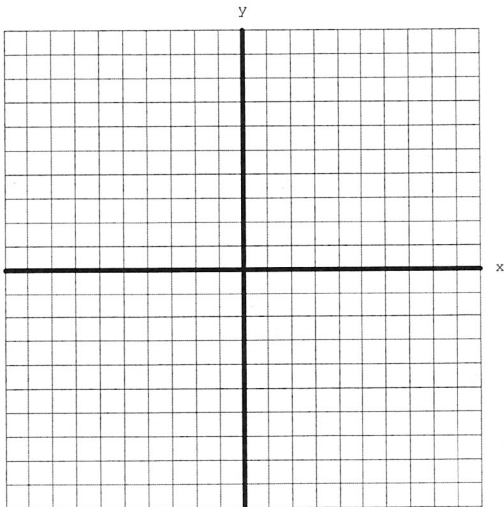
c)  $2x^2 - 7x = 15$

d)  $-3x^2 - 4x + 15 = 0$

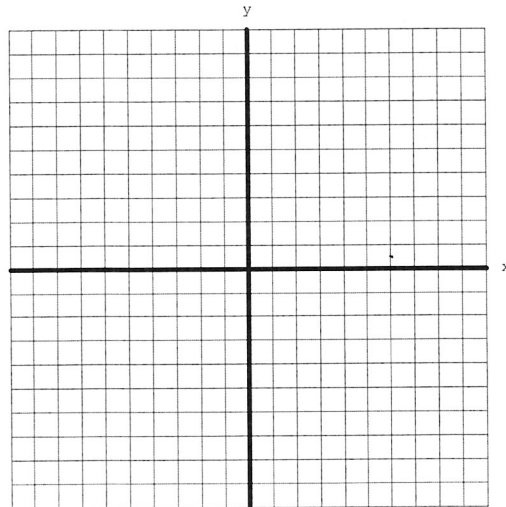
e)  $x(x + 3) = 2x(x + 5) + 1$

6. Sketch the following quadratic functions:  
Show your work.

a)  $y = x^2 + 3x - 10$



b)  $y = 2x^2 - 4x - 3$



7. Kelly kicked a soccer ball in such a way that the relationship between the height  $h$  of the ball in metres and the elapsed time  $t$  in seconds, from the instant of kicking the ball, can be expressed by the equation:

$$h = -4.9t^2 + 28t + 0.5$$

(Draw a diagram to help you.)

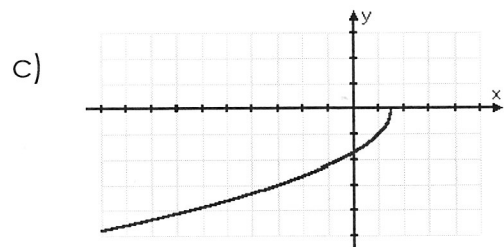
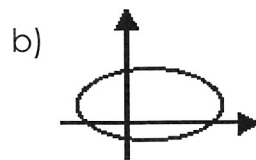
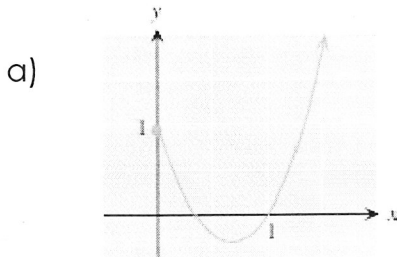
a) Determine the maximum height of the ball?

b) How long is the ball in the air?

8. a) Graph:  $y = 2x^2 - 3$ .  
b) Graph the equation  $y = \frac{1}{2}x^2 + 3$ .  
c) At what two points do the graphs intersect?

UNIT 4 – Transformation of Functions

1. Which of the following graphs are functions?



2. State true or false for each:

- a) If  $f(x) = x^2 + 3x$ , then  $f(-1) = -4$ .  
b) If  $f(x) = 2x^2 + 1$ , then  $f(3) = 37$ .  
c) Some functions assign two or more values to certain numbers in their domains.

3. Given  $f(x) = x^2 + 2x - 1$ , find  $f(2)$ .

4. Given  $f(x) = x^2 + 2x - 1$ . Evaluate  $f(-2)$ .

5. Graph  $y = x^2$ ,  $y = 2x^2$ ,  $y = \frac{1}{2}x^2$ .

State how each is similar and different to  $y = x^2$ . State the mapping for each in relation to  $y = x^2$ .

6. Graph  $y = x^2$ ,  $y = 2(x^2 - 4)$ ,  $y = x^2 - 4$ .

State the mapping for each in relation to  $y = x^2$ . State the domain and range.

7. Find the inverse of  $f(x) = 6x - 7$

8. Graph  $y = \sqrt{x}$  and  $y = 2\sqrt{x-4} + 3$ . State the mapping and the domain and range.

**9. Fill in the blanks of the following chart:**

Equation	Vertex	Number of x-intercepts	Transformations starting from $y = x^2$	Domain and Range
$y = 3(x - 2)^2 - 1$				
$y = -\frac{1}{3}(x - 2)^2 + 5$				
$y = x^2 - 25$				
$y = \frac{1}{2}(x - 5)^2$				

UNIT 5 – Exponential Functions

1. Without the use of a calculator evaluate each expression.

a)  $4^{-2}$                       b)  $5^{-5} \div 5^3$     c)  $3^2 \times 3^3$

d)  $(2^2)^{-4}$                       e)  $\frac{4^{-2} \times 4^4}{4^{-5} \times 4^3}$                       f)  $\frac{3^2 \times 4^5}{4^6 \times 3}$

g)  $81^{\frac{1}{4}}$                       h)  $16^{\frac{3}{4}}$                       i)  $(-27)^{\frac{4}{3}}$                       j)  $16^{\frac{-3}{4}}$