Exam Review
Transformations of Functions

1a) → each x value has only one corresponding y value

b) → use vertical line test; the line should cross no more than one point

c) → sub in values for x should only result in one value for y

2. a) $f(5) = 2(5) - 3$
$= 10 - 3$
$= 7$

b) $g(x+1) = 4(x+1)^2 - 5$
$= 4(x+1)(x+1) - 5$
$= 4(x^2 + 2x + 1) - 5$
$= 4x^2 + 8x + 4 - 5$
$= 4x^2 + 8x - 1$

c) $3f(x) + 7 = 3(2x-3) + 7$
$= 6x - 9 + 7$
$= 6x - 2$

d) $f(g(x)) = 2(4x^2 - 5) - 3$
$= 8x^2 - 10 - 3$
$= 8x^2 - 13$

3. From the previous question, $f(g(x)) = 8x^2 - 13$

$g(f(x)) = 4(2x-3)^2 - 5$
$= 4(2x-3)(2x-3) - 5$
$= 4(4x^2 - 12x + 9) - 5$
$= 16x^2 - 48x + 36 - 5$
$= 16x^2 - 48x + 31$

Clearly $f(g(x)) \neq g(f(x))$.

4. a) $f(x) = 10x - 7$

$y = 10x - 7$

$10x - 7 = y$

$\frac{10x}{10} = \frac{y+7}{10}$

$x = \frac{y+7}{10}$

$f^{-1}(x) = \frac{x+7}{10}$

b) $f(x) = 3x^2 - 5$

$y = 3x^2 - 5$

$3x^2 - 5 = y$

$\frac{3x^2}{3} = \frac{y+5}{3}$

$\sqrt{x^2} = \pm\sqrt{\frac{y+5}{3}}$

$x = \pm\sqrt{\frac{y+5}{3}}$

$f^{-1}(x) = \pm\sqrt{\frac{x+5}{3}}$

#5 a) $y = 2\sqrt{x-1} - 3$

$k = 1$
$p = 1$
$a = 2$
$q = -3$

x	$y = \sqrt{x}$
0	0
1	1
4	2
9	3

See graph on next page.

Domain $= \{x \mid x \geq 1, x \in \mathbb{R}\}$
Range $= \{y \mid y \geq -3, y \in \mathbb{R}\}$

b) $y = -|2x+4| + 5$

$y = -|2(x+2)| + 5$

$k = 2$
$p = -2$
$a = -1$
$q = 5$

x	$y = \lvert x \rvert$
-2	2
-1	1
0	0
1	1
2	2

See graph on next page.

Domain $= \{x \mid x \in \mathbb{R}\}$
Range $= \{y \mid y \leq 5, y \in \mathbb{R}\}$

#6 $y = -3f(-4x - 8) + 9$

$y = -3f(-4(x+2)) + 9$

→ horizontally compressed by a factor of 4, horizontally reflected about the y-axis
→ shifted left 2 units
→ expanded vertically by a factor of 3, vertically reflected about the x-axis
→ shifted up 9 units

Exam Review: Transformation of Functions

1. Describe how it can be determined if a relationship is a function from:
   a. Table of values
   b. Graph
   c. Equation

2. Given f(x) = 2x – 3 and g(x) = 4x^2 – 5, evaluate or expand and simplify the following:

   a) f(5) b) g(x+1) c) 3f(x)+7 d) f(g(x))

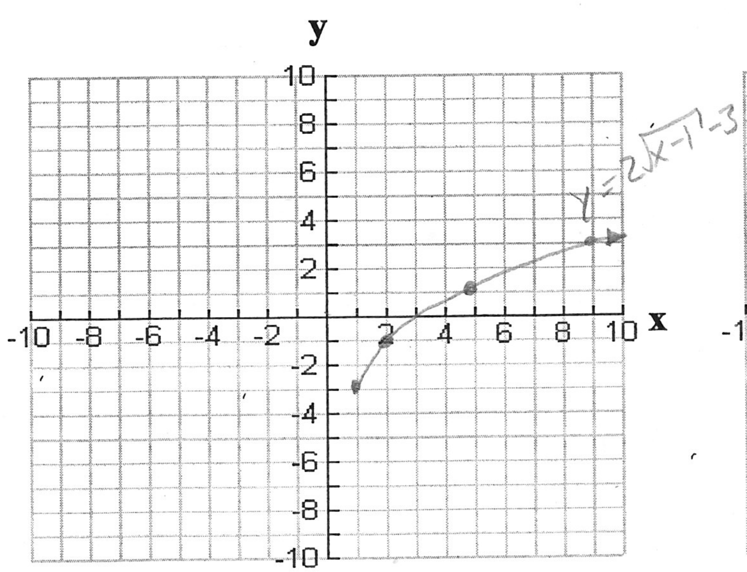
3. Does f(g(x)) always equal g(f(x))? Explain using an example.

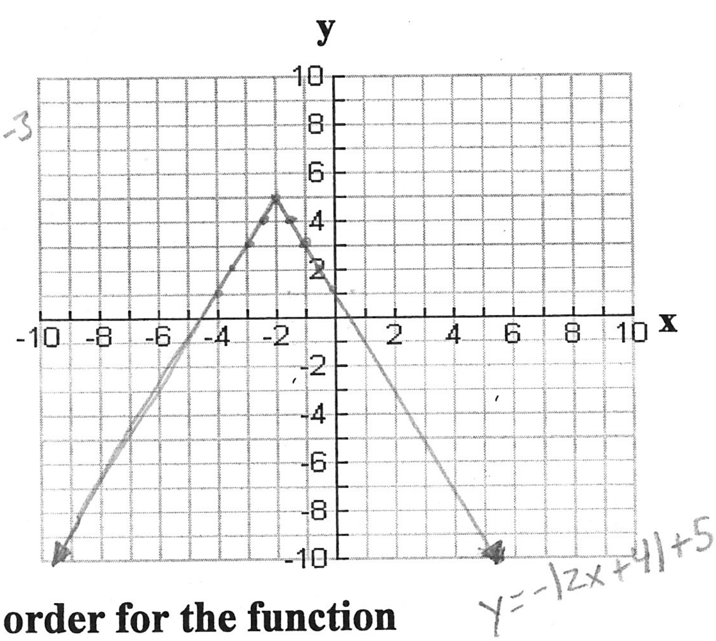
4. Given f(x), determine f^{-1}(x):

   a) f(x) = 10x – 7 b) f(x) = 3x^2 - 5

5. Graph the functions below using transformations and state the domain and range.

   a) $y = 2\sqrt{x-1} - 3$ b) $y = -|2x+4| + 5$





6. Describe the transformations in order for the function
   y = -3f(-4x - 8)) + 9