

Hmwk: pg 490 # 1, 2ab, 5, (6), 8, 11, (12)
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Simple Interest Revisited

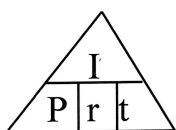
For simple interest, we are given:

$$A = P + I$$

where

- A is the future value of the account and interest
- P is the principal (initial amount invested)
- I is the total interest earned

Equations that relate the total interest, principal, interest rate and time can be derived from the simple interest triangle as follows:



$$I = Prt, \quad P = \frac{I}{rt}, \quad r = \frac{I}{Pt}, \quad t = \frac{I}{Pr}$$

Example 1

Mrs. Gordon invests \$2500 in an account that collects 4% simple interest annually. How much is the investment worth in 5 years?

$$\begin{array}{l} I = ? \\ P = 2500 \\ r = 0.04 \\ t = 5 \end{array} \quad \begin{array}{l} I = Prt \\ = 2500(0.04)(5) \\ = 500 \end{array} \quad \begin{array}{l} A = P + I \\ = (2500) + (500) \\ = \text{\$ } 3000 \end{array}$$

Example 2

Mr. Eyehartmath invests \$3000 into a simple interest account that grows to \$4500 in 10 years. What was the annual interest rate?

$$\begin{array}{l} A = P + I \\ 4500 = 3000 + I \\ I = 1500 \end{array} \quad \begin{array}{l} r = \frac{I}{Pt} \\ = \frac{1500}{3000(10)} \\ = 0.05 \\ = 5\% / \text{year} \end{array}$$

With simple interest accounts, the interest earned is NOT reinvested. As such the interest earned each year is the same and the combined value of the account and interest grows linearly. Most investments do not operate this way.

Compound Interest

With compound interest, the interest earned after one term is reinvested so that interest earned for subsequent terms is made on the sum of the principal and any previous interest that was earned.

The following equation will be used for compound interest:

$$A = P(1 + i)^n$$

where

- A is the future value of the investment
- P is the principal
- i is the interest that was earned
- n is number of ~~the number of~~ times interest is collected.

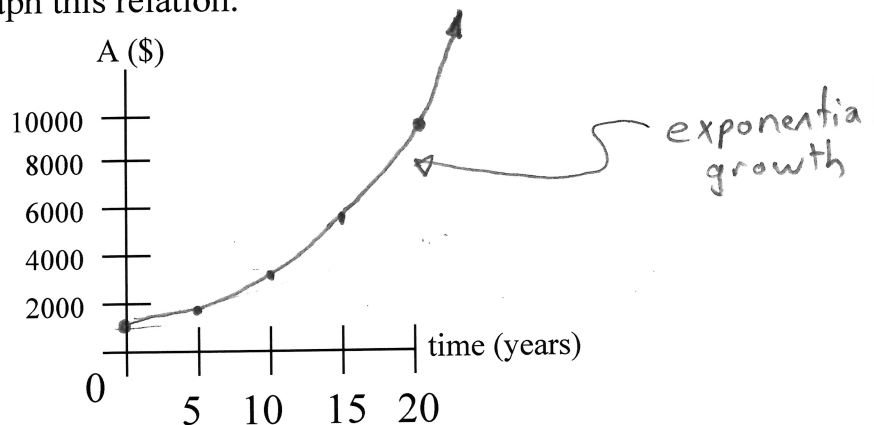
Example 1

George invests \$1000 that earns 12% interest compounded annually.

a) Complete the table of values for the future value of the investment over 20 years.

time (yrs)	value of the investment $A = P(1 + i)^n$
0	1000
5	$1000(1 + 0.12)^5 = \$1762.34$
10	$1000(1.12)^{10} = \$3105.85$
15	$1000(1.12)^{15} = \$5473.57$
20	$1000(1.12)^{20} = \$9646.29$

b) Graph this relation.



Example 2

How much money must Uncle Leo put into an investment that earns 8% interest compounded annually for it to be worth \$10000 in 5 years? Create an equation for 'P' then use it to solve this question.

$$A = \frac{P(1+i)^n}{(1+i)^n}$$

$$P = \frac{A}{(1+i)^n}$$

$$A = 10000$$

$$P = ?$$

$$i = 0.08$$

$$n = 5$$

$$P = \frac{A}{(1+i)^n}$$

$$= \frac{10000}{(1.08)^5}$$

$$= \$6805.83$$

Often, interest is compounded more often than once a year.

Compound Period	Interest Rate, i	Number of compounding periods, n
annually	unchanged	unchanged
semi-annually	divide annual interest by 2	multiply years by 2
quarterly	divide annual interest by 4	multiply years by 4
monthly	divide annual interest by 12	multiply years by 12

Example 3

8% / a

Sandra invests \$1000 in a mutual fund that earns 8% compounded quarterly. How much is it worth after 5 years?

$$A = ?$$

$$P = 1000$$

$$i = 0.08 \div 4 = 0.02$$

$$n = 5 \times 4 = 20$$

$$A = P(1+i)^n$$

$$= 1000(1.02)^{20}$$

$$= \$1485.95$$

Example 4

Shauna wants to have \$10000 in the bank in 4 years for post secondary education. How much must she invest today at 6% compounded monthly?

6% / a

$$A = 10000$$

$$P = ?$$

$$i = 0.06 \div 12 = 0.005$$

$$n = 4 \times 12 = 48$$

$$P = \frac{A}{(1+i)^n}$$

$$= \frac{10000}{(1.005)^{48}}$$

$$= \$7870.98$$