Elimination of Common Factors in Rational Expressions

Consider the following rational expression:

$$\frac{\cancel{2}(3x-5)}{\cancel{2}x} \rightarrow \frac{3x-5}{x}$$

In this example, the common factor '2', located in both the numerator and denominator, can be cancelled/eliminated since both can each be factored out to create a single multiplier of one in front as follows:

$$\frac{2(3x-5)}{2x}$$
$$= \frac{2}{2} \cdot \frac{3x-5}{x}$$
$$= 1 \cdot \frac{3x-5}{x}$$
$$= \frac{3x-5}{x}$$

Sometimes, there are situations where this type of cancelling process appears to be an option when, in fact, it is not. For example:

$$\frac{x+y}{x} \neq \frac{x}{x} \cdot \frac{+y}{1}$$

In this example, the presence of 'x' at the beginning of both the numerator and denominator create the illusion that they can be cancelled, however, this is not an option since the x in the numerator cannot be factored out due to the presence of the addition operator.

It is also possible to cancel common factors when the multipliers are present on opposite sides of two fractions that are being multiplied. For example, apply a cancelling technique to the following:

$$\frac{2x}{3y-1} \cdot \frac{3y}{4x-6}$$
$$= \frac{\cancel{2}x}{3y-1} \cdot \frac{3y}{\cancel{2}(2x-3)}$$
$$= \frac{2(3)xy}{2(3y-1)(2x-3)}$$
$$= \frac{2}{2} \cdot \frac{3xy}{(3y-1)(2x-3)}$$
$$= 1 \cdot \frac{3xy}{(3y-1)(2x-3)}$$
$$= \frac{3xy}{(3y-1)(2x-3)}$$

Examples:

Use the cancelling process (if possible) to simplify each rational expression below.

a) $\frac{2x+8}{6}$

$= \frac{\cancel{2}(x+4)}{\cancel{2}(3)}$

$= \frac{x+4}{3}$

b) $\frac{9x-3y}{6}$

$= \frac{\cancel{3}(3x-y)}{\cancel{3}(2)}$

$= \frac{3x-y}{2}$

c) $\frac{3xy+x}{2x}$

$= \frac{\cancel{x}(3y+1)}{\cancel{x}(2)}$

$= \frac{3y+1}{2}$

d) $\frac{4x}{8x^2-2}$

$= \frac{\cancel{2}(2x)}{\cancel{2}(4x^2-1)}$

$= \frac{2x}{4x^2-1}$

e) $\frac{5x+y}{10xy}$

→ can't cancel

f) $\frac{2x}{3x-5} \cdot \frac{4y-1}{2x-10}$

$= \frac{\cancel{2}x}{3x-5} \cdot \frac{4y-1}{\cancel{2}(x-5)}$

$= \frac{x(4y-1)}{(3x-5)(x-5)}$

g) $\frac{3x}{\boxed{2x-8}} - \frac{\boxed{2}}{9x-15}$

=

prevents the cancelling

h) $\frac{x^2-\overset{2}{\cancel{4}}x-24}{x-6}$

$= \frac{\cancel{(x-6)}(x+4)}{\cancel{(x-6)}}$

$= x+4$

Homework: Complete questions below + pg 105 # 1, 2, 5, 6, 7, 8, 9

Cancel/Eliminate common factors from the numerator and denominator (if possible).

a) a) $\frac{3x-12}{9}$

$= \frac{\cancel{3}(x-4)}{\cancel{3}(3)}$

$= \frac{x-4}{3}$

b) $\frac{2x-4}{6x}$

$= \frac{\cancel{2}(x-2)}{\cancel{2}(3x)}$

$= \frac{x-2}{3x}$

c) $\frac{12y-7xy}{3y}$

$= \frac{\cancel{y}(12-7x)}{3\cancel{y}}$

$= \frac{12-7x}{3}$

d) $\frac{9x}{3x^2+3}$

$= \frac{\cancel{3}(3x)}{\cancel{3}(x^2+1)}$

$= \frac{3x}{x^2+1}$

e) $\frac{x-2y}{4xy}$

= can't cancel

f) $\frac{3x}{5x-15} \cdot \frac{10y-20}{x+8}$

$= \frac{3x}{\cancel{5}(x-3)} \cdot \frac{\cancel{5}(2)(y-2)}{x+8}$

$= \frac{6x(y-2)}{(x-3)(x+8)}$

g) $\frac{x}{y+1} + \frac{y}{x(x-3)}$

can't cancel

h) $\frac{x+3}{x^2-9}$

$= \frac{\cancel{x+3}}{(x-3)\cancel{(x+3)}}$

$= \frac{1}{x-3}$